God, Belief and Explanation

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THE LONDON SCHOOL OF ECONOMICS AND POLITICAL SCIENCE

God's Order, Man's Order and the Order of Nature

DEDICATION

As always, the late Jennifer Redhead provided constant support and encouragement.

God's Order, Man's Order and the Order Of Nature
ISSN 2045-5577

CONTENTS

MOTTO

There is grandeur in this view of life, with its several powers, having been originally breathed by the Creator into a few forms or into one; and that, whilst this planet has gone cycling on according to the fixed law of gravity, from so simple a beginning endless forms most beautiful and most wonderful have been, and are being, evolved.

'On the origin of species by means of natural selection'

Charles Darwin, 2nd edition, 1860

ABOUT THE BOOK

This is a book about science and religion. It is aimed mainly at committed atheist Richard Dawkins and his philosophical counterpart Dan Dennett.

We acknowledge the support of the Leverhulme Trust who gave an Emeritus Fellowship to Michael Redhead to make the book possible.

The plan of the book is as follows:

In **Chapter One**, Frontispiece, we review briefly the works around which our own book revolves. We list some important books that explain the role of religion, and also those which adopt the atheist stance.

Chapter Two is entitled The Unseen World. This is our own view of what philosophy of science can teach us.

Chapter Three deals with our own view of what religion means for us. It is entitled Religion and Reason. We have two types of knowledge, the laws given us by science and the role of God's law in the moral sphere. Religion propounds the view that God's Will is what makes moral certainty effective. We have two sorts of law and religion enters in the second sense. But religion offers us a guide to the moral law. We can accept the moral law as our best intention to meet the demands of religion. Now what about the role of reason? Reason applies the rational power of the mind. According to St Thomas Aquinas's grace enables us to go beyond reason to explore the mysteries of faith. How does reason work? There are many areas where faith impinges on the world of science, and that is where belief in science acts back in relation to faith-centred belief. In other words the role of doctrine is not immune to the demands of reason. This is the role Vatican II's demand on how faith should be altered to agree with the facts of science. So, we have our own faith centred on reason. Faith extends out beyond reason, but there are many varieties of faith that fit with reason. We want to argue that our view of faith involves a system of cultural, historical and other textual knowledge that makes sense to us, but of course, we must recognise that other faiths are available. So we accept our own faith, but admit that other faiths make equal sense to other people. Thus we accept our personal faith, but freely accept that we have no certainty that we can know the supreme reality that rules everything.

Chapter Four is concerned with what we can learn from Gödel's theorem.

In **Chapter Five** we look at quantum mechanics and what the nonlocality arguments really mean.

In **Chapter Six** we deal with so-called revealed religion. This is a view defended by fundamentalists, which seeks to define its own version of theism, and attacks all other views as heretical. The reason for the bad press of religion lies in the sense of my being right, and that other religions are false, and must be radically evangelised. We reject this view of revealed religion and in place explain our own view of religion as tied to a reasoned view of what the moral view entails.

Finally in **Chapter Seven** we provide a very brief epilogue of our conclusions.

We are grateful to John Lucas, Richard Sorabji, Patrick Hobson, Arnie Koslow, Keith Ward and Nancy Cartwright for their comments. We also acknowledge the help of Seamus Bradley with the index. We are also grateful for comments by members of the LSE's Order Project funded by the Templeton Foundation.

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Michael Redhead obtained his PhD in mathematical physics at UCL. He was appointed Professor of Philosophy of Physics at King's College London, and was then appointed to the Chair of History and Philosophy of Science at Cambridge. On retiring from Cambridge he was elected to be the Co-Director of the Centre of Natural and Social Science at LSE. Furthermore he was also elected as a Fellow of The British Academy.

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ACKNOWLEDGEMENTS

We are grateful to the editors C. Cheyne & John Worrall for allowing us to use material in their book *Rationality and Reality: Conversations with Alan Musgrave*, also to the editor of the British Journal for the Philosophy of Science for the material published as *The Mathematics of the Mind*. We thank R. J. Russell, P. Clayton, K. Wegster-McNelly and J. Polkinghorne for permission to use the article 'The Tangled Story of Nonlocality in Quantum Mechanics' in their book *Quantum Mechanics: Scientific Perspectives on Divine Action*.

Fig 1 A nineteenth-century woodcut that supposedly presents the medieval view of the Universe. Beyond the sphere of stars lie the celestial machinery and other wonders. See Fig.5.1 on p.101 of Edward R Harrison, *Cosmology: The Science of the Universe*, Cambridge: Cambridge University Press, 1981.

Fig 2 The Moon as seen by Galileo in 1609. Fig.106, p.243 Charles Singer, *A Short History of Scientific Ideas to 1900*, Oxford: Clarendon Press, 1959.

Fig 3 Stellati's figure of a bee of original size (1629), Fig.108, p.255, Charles Singer, *A Short History of Scientific Ideas to 1900*, Oxford: Clarendon Press, 1959.

Fig 4 Spermatazoa as seen in the seventeenth century- Hartsoeker (1694), Fig.119, p.287, Charles Singer, *A Short History of Scientific Ideas to 1900*, Oxford: Clarendon Press, 1959.

Fig 5 The 80 inch (2 metre) liquid hydrogen bubble chamber at the Brookhaven laboratory (USA). The chamber itself is hidden by the yoke of the electromagnet. The particles enter at the level of the second person on the left. Cameras are installed where the personnel are standing on the right. See Fig.87, p.244, Robert Gouiran, *Particles and Accelerators*, World University Library: George Weidenfeld and Nicolson, 1967.

Fig 6 A K- meson enters at the bottom of a large liquid hydrogen chamber at the Brookhaven laboratory. It interacts with a proton and produces an W^-,a K° and a K^+. See Robert Gouiran, Fig 18, p.87, ibid.

Fig 7 We are grateful to the librarian, David Allan, of the Horniman Museum, for permission to use a photograph of the Tibetan Ghost Trap.

CHAPTER 1 – FRONTISPIECE

There is at present a great debate between the religious believers[1] who see some version of the rôle of God as determining the moral framework of human life, and the atheists[2] who regard the religious view as a pathetic fabrication. The aim of the present work is to straighten out the argument, so that we can appreciate how we can understand the religious view, and see what that view really amounts to.

We object to the following. The task of creation science is to set God's law, not just in the moral sphere, but as an abiding fact of life in all aspects of creation. They rely on the book of Genesis as the God-given record of how creation occurs.

An American Seventh-Adventist geologist, George McCready Price, published in 1906 his book' Illogical Geology: The Weakest Part in the Evolution Theory', and in more detail ' New Geology' [1923]. This explained geological evidence by a recent universal deluge [Noah's flood].

This led to the Creation Science movement. Henry M. Morris, a Texan Baptist teacher of civil engineering, founded the Creation Science Society in 1963.

A new departure was the Intelligent Design programme. For example, Michael Behe in his 1996 book 'Darwin's Black Box – The Biochemical Challenge to Evolution' cites the case of the bacterial flagellum as an example of irreducible complexity. This is a small, hair-like apparatus with a complex molecular motion, used by some bacteria to propel themselves. It is composed of dozens of separate proteins, all of which must work in concert for the hair-like 'propeller' to move. But this has not been considered by biologists as a viable case. See Ronald L. Numbers, 2006, 'The Creationists: From Scientific Creationism to Intelligent Design', expanded edition. The Scope's trial of 1925 was an attempt to block evolutionism in school teaching. For a critical defence of Scope see Phillip Kitcher 'Abusing Science: The Case against Creationism', 1982. See also John Brockman (ed) 'Intelligent Thought: Science versus the Intelligent Design Movement', 2006.

In brief, we believe in the Darwinian theory of evolution, but that does not mean we deny religion.

ENDNOTES

1. **VIEWS IN FAVOUR OF RELIGION**

Alister McGrath, 2005, *Dawkins' God: Genes, Memes and the Meaning of Life*, Oxford: Blackwell. A detailed reply with many references to Dawkin's views on religion. Formerly Professor of Historical Theology at Oxford, now Professor of Theology, King's College London.

Michael Ruse, 2005, *The Evolution – Creation Struggle*, Cambridge: Harvard University Press. Ruse is Professor of Philosophy, Florida State University. His book examines the evolution-creation struggle with many illuminating details.

Owen Gingerich, 2006, *God's Universe*, Massachusetts: Harvard University Press. A Professor of Astronomy and of History of Science at Harvard supports a meeting place for science and religion.

John Cornwall, 2007, *Darwin's Angel: An Angelic Repost to the God Delusion*, London: Profile. A gentle response to The God Delusion setting out the reply of a Guardian Angel in defence of religion.

Peter J. Bowler, 2007, *Monkey Trials and Gorilla Sermons: Evolution and Christianity from Darwin to Intelligent Design*, Cambridge: Harvard University Press. A reasoned account of science and religion by an historian of science, Professor of History of Science, Queens University, Belfast

John C. Lennox, 2007, *God's Undertaker: Has Science buried God?,* Oxford: Lion Hudson. Reader in Mathematics at Oxford, puts forward the case for religion.

Keith Ward, 2008, *Why There Almost Certainly Is a God-Doubting Dawkins,* Oxford: Lion Hudson. A refutation of Dawkin's book The God Delusion by the Emeritus Professor of Divinity at Oxford.

John F. Haught, 2008, *God and the New Atheism: A Critical Response to Dawkins, Harris, and Hitchens.* Louisville: Westminster John Knox Press. Haught argues for the subjective view favoured by religion, as against the objective view given by science, which is flawed by circularity. He was formerly Professor of Theology at Georgetown University.

John Polkinghorne and Nicholas Beale, 2009, *Questions of Truth*, Louisville Kentucky: Westminster John Knox Press. John Polkinghorne is an ordained priest, and a Fellow of the Royal Society. In this book a series of questions about God, science and belief are addressed. Polkinghorne was formerly Professor of Mathematical Physics at Cambridge.

2. **VIEWS AGAINST RELIGION**

Richard Dawkins, 1976 (2nd ed 1989) *The Selfish Gene*, Oxford: Oxford University Press. The original presentation of Darwin's ideas. Dawkins is the Emeritus Charles Simonyi Professor for the Public Understanding of Science, at Oxford.

Richard Dawkins, 1986, *The Blind Watchmaker: Why the Evidence of Evolution Reveals a Universe Without Design*, Harlow: Longman Scientific. In this work Dawkins makes the point that Darwinian evolution is without purpose.

Richard Dawkins, 1995, *River out of Eden: A Darwinian View of Life*, London: Phoenix. Dawkins uses the metaphor of a river to represent the flow of information through time. Evolution rather than God is the key to this story.

Richard Dawkins: 1996, *Climbing Mount Improbable*, London: Viking. Based on his Royal Institution Christmas lectures, Dawkins considers the way evolution describes the combination of perfection and improbability in the process of living things.

Richard Dawkins, 1998, *Unweaving the Rainbow: Science, Delusion and the Appetite for Wonder*, Boston: Houghton Miflin. Dawkins here embarks on a wider field that includes the human brain. It represents Keats's view of Newton's work destroying the poetry of the rainbow, but argues that evolution inspires the human imagination and enhances our wonder of the world.

Richard Dawkins, 2003, *A Devil's Chaplin: Selected Essays*, London: Weidenfeld and Nicholson. In selected essays Dawkins writes on evolution, education, justice, history of science and of course religion. Argues well with his characteristic verve.

Richard Dawkins, 2004, *The Ancestor's Tale: A Pilgrimage to the Dawn of Evolution*, New York: Weidenfeld and Nicholson. Dawkins refers to the work of Chaucer's Canterbury Tales in tracing back the history of evolution, arriving at the origin of life itself.

Richard Dawkins, 2006, *The God Delusion*, Boston: Houghton Miflin.This is the famous [or infamous] book attacking religion.

Richard Dawkins, 2009, *The Greatest Show on Earth: The Evidence for Evolution*, Transworld Publishers: Random House. Argues for the factual nature of evolution.

Daniel D. Dennet, 1995, *Darwin's Dangerous Idea: Evolution and the Meaning of Life*, New York: Simon and Schuster. A philosophical commentary on the idea of evolution and its overview of Darwinian theory. Dennett is Professor of Distinguished Arts and Sciences at Tufts University.

Daniel D. Dennet, 2006, *Breaking the Spell: Religion as a Natural Phenomenon*, New York: Viking. Dennet argues for religion as a natural phenominon.

Sam Harris, 2004, *The End of Faith: Religion, Terror, and the Future of Reason*, New York: W.W. Norton and Co. Harris argues for the dangers of organised religion, and attacks the notion of tolerance.

Christopher Hitchens, 2007, *God is not Great*, London: Atlantic Books. Another attack on religion.

Jerry A. Coyne, 2009, *Why Evolution is True*, Oxford: Oxford University Press. A delightful overview of modern evolutionary theory, showing why scientists believe it to be true. Coyne is the Professor of Ecology and Evolution at the University of Chicago.

CHAPTER 2 – THE UNSEEN WORLD

Science deals with many things we cannot *directly* observe[1]. By directly is meant with the unaided senses. For example there are the elementary particles such as electrons and quarks which are supposed to provide the microscopic building blocks of matter, but also the mysterious photons and gluons etc. which mediate interactions between the microscopic building blocks. And then of course in molecular biology there are the proteins and genes and so on which explain the processes underlying living organisms. But also there are more abstract entities such as energy and entropy which are not part of our immediate sensory experience, and still more abstract entities, like numbers and mathematical points, not just indeed in physical space, but in still more abstract mathematical spaces, such as Hilbert space in quantum mechanics.

So much of modern science seems concerned with what can be called the Unseen World (using sight as a generic term covering all the senses). Indeed the Unseen World effectively constitutes what we may call the scientific world-view. This was famously illustrated by Eddington with his talk of 'the two tables', the table of everyday experience, firm and solid in front of him, and the scientific table, mostly empty space permeated by the force-fields of elementary particles. Which is the `real' table? And which is the true story, the scientific story or the everyday story?

In this chapter we shall explore the cognitive credentials of the Unseen World from both an historical and a modern perspective. Hume famously warned that 'the ultimate springs and principles are totally shut up from human curiosity and enquiry' But science seems not to have heeded Hume's warning, and let me begin by reminding you of a famous medieval woodcut, in which a curious person peers beyond the vault of the heavens to learn of the hidden mechanisms and contraptions that lie beyond.

Medieval Woodcut illustrating the hidden mechanisms of the Universe

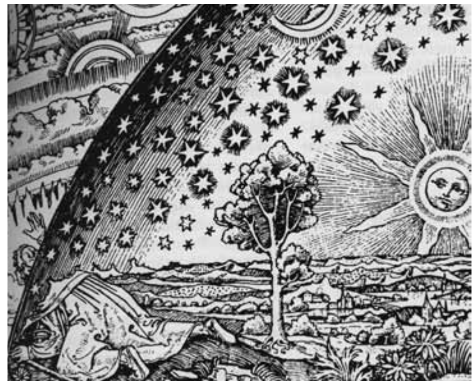

Fig 1

But the question is: what *can* we directly observe with the unaided senses? Microscopic objects in our immediate vicinity perhaps, such as the table in front of me or the chair next to me. But is it the table we see, or the light reflected off the table, or is it the electrical stimulation in the retina caused by the light, or in the optic nerve, or what is it exactly that we see?

Naively we can think of a sort of homunculus inside our brains (our conscious selves) reading out and interpreting the input signals, but if our brains (and minds?) are just part of nature, then the whole idea of a homunculus, or the ghost in the machine, as the philosopher Gilbert Ryle called it, seems patently absurd. This is the problem of consciousness, but it is not the problem which is going to be considered here, interesting and important though it is.

Let us start with the assumption that we do, in some sense, see tables and chairs in a good light possessing normal eyesight and so on. Even if we don't actually see them, ie, they are not actually *being* observed, nevertheless they are *observable* in the sense that it is *possible* to see them.

Some philosophers of science, and indeed historically many scientists, have thought that science is concerned with discovering regularities in the behaviour of observable entities. Such people are generally called positivists. Scientific knowledge can be checked out in a positive fashion by direct observation. Labels such as `positivist', and more particularly its cognate 'empiricist', are used with many shades of meaning in philosophy. We shall use such terms with a broad brush, just to give the general idea.

At first blush the positivist position sounds attractive. The scientific attitude has progressed by getting rid first of supernatural spirits and gods controlling the world, then of theoretical metaphysical concepts like dormitive virtues and other mysterious substantial forms beloved of the Aristotelians, and finally arriving at the culmination of what the nineteenth century philosopher Auguste Comte called positive (i.e. non-speculative) knowledge.

But has science really followed the positivist programme? There are all kinds of difficulties. If we are restricted to direct observation then what is the point of scientific instruments like telescopes and microscopes? Surely these are supposed to enable us to see things that we can't directly observe?

There is a significant difference here between the telescope and the microscope. The optical telescope enables us to see things that we could see directly if we were differently located, i.e. moved closer to the distant tower or close up to the moons of Jupiter or whatever. But for the microscope it is not a matter of relocating ourselves. For the virus or the cell to become directly visible to us we would have to change our normal sensory apparatus or adopt the perspective of the Incredible Shrinking Man. So to count the virus or the cell as observable needs rather more science fiction than the case of the telescope.

Historically the first practical versions of the telescope and the compound microscope were employed by Galileo at the beginning of the seventeenth century. The telescope revealed all sorts of oddities in the heavens, from mountains on the moon to the satellites of Jupiter, announced by Galileo in his famous book *The Starry Messenger* (1610).

The Moon, as seen by Galileo, 1610

Fig 2

What was the reaction of Galileo's Jesuit opponents? Some refused even to look through the telescope, averring that if God had intended us to inspect the heavens so closely he would have equipped us with telescopic eyes! Others claimed Galileo's observations were artefacts of the instrument.

With the microscope, amazing detail was exposed; for example, the famous drawing dated 1625 of a bee, made by Francesco Stelluti looking through an early microscope.

The Figure of a Bee – Francesco Stelluti, 1625

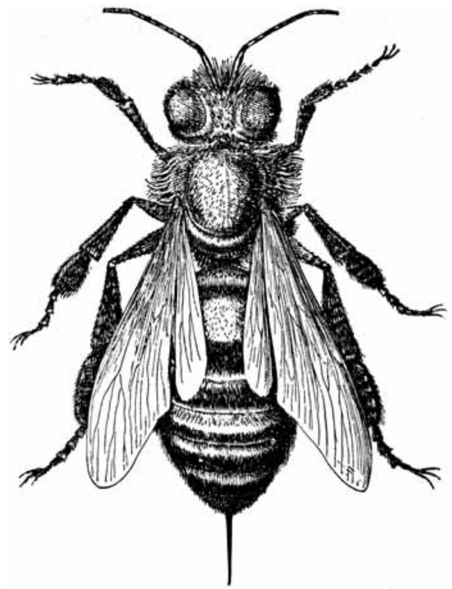

Fig 3

But sometimes people saw what they wanted or expected to see. Preformationists, like Nicholaas Hartsoeker, in embryology at the end of the seventeenth century claimed to see the homunculus sitting perfectly preformed in the head of the spermatozoon!

Spermatazoon, Nicholaas Hartsoeker, 1694

Fig 4

What we see is largely determined by the overall theoretical background of our thinking. The slogan here is the theory-ladenness of observation. We have already had occasion to question whether the table or chair is *directly* observable. Is not observation always a case of probing or interacting with the physical world, and don't we always observe things by the *effects* they produce ultimately in our conscious minds? We often talk loosely of observing fundamental particle reactions, for example, with a bubble chamber or suchlike, but it's only when we *look* at the photographic plate recording the tracks that the observation is translated into positive knowledge for *us*. Compare the discovery of the Ω^- particle.

Bubble Chamber, Brookhaven, 1964

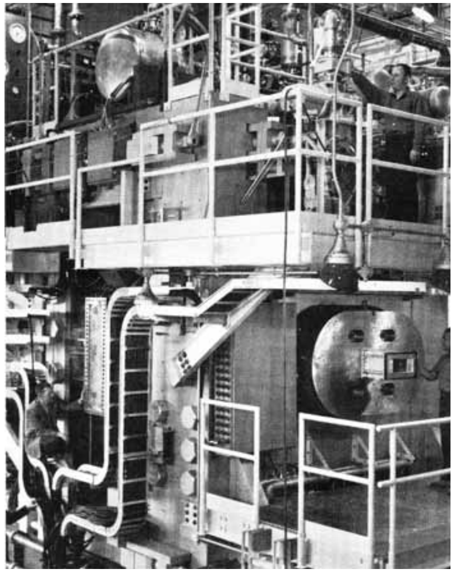

Fig 5

Discovery of the Ω^- Particle, 1964

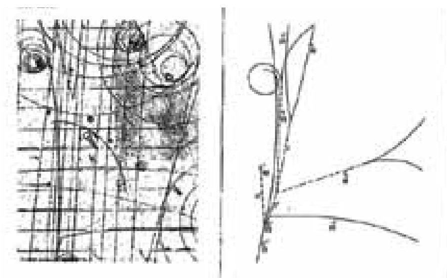

Fig 6

From this perspective, electrons, quarks, genes and viruses are after all observable. So do they really belong to the Unseen World, and on *that* account should they be eschewed by the scientist? This debate was carried on particularly vigorously at the end of the nineteenth century in respect of the reality of atoms. For Mach, Ostwald and others, the atoms of the physicist and the chemist were just fictional entities introduced as speculative mechanisms for explaining empirical regularities about chemical combination or the properties of gases. They were not to be thought of as 'real' in any robust philosophical sense.

To the modern scientist it is usually assumed that these debates have long been settled in favour of a realist conception of so-called theoretical entities rather than their positivist dismissal. But again things are less simple than they seem.

If we look at the history of science we can see it as a series of U-turns about the explanatory theoretical structures that lie behind or beneath the world of macroscopic experience. Entities like phlogiston or the luminiferous aether or caloric have simply disappeared from the scientific vocabulary and the nature of atoms and molecules is quite different from the modern perspective of quantum mechanics than from the billiard ball conception of the nineteenth century. This leads to the famous pessimistic induction. If we have been so often wrong in the past, is it not pure hubris to believe that our present scientific theories won't look equally ridiculous a hundred years from now?

To defuse the pessimistic induction philosophers have tried to read the history of science in a more continuous and progressive fashion.

It has been argued by John Worrall (1989), for example, that although the ontology of physical theory changes abruptly, nevertheless there may be what might be called structural continuity in the sense that in many cases the mathematical equations survive. Only the *interpretation* of the quantities entering into the equations changes. There are two versions of this structuralist philosophy. In an extreme, even bizarre, ontological version, it is only structure which really exists. Everything else is just imaginative fiction. In a more prosaic epistemic version, structure is all that we can claim reliably to *know*. We don't deny that atoms or quarks exist, just that we never know what their true natures are, only the *mathematical* description of how they are constructed, related to one another, behave in various experimental contexts and so on. The basic argument here is that the continuity of mathematical structure defeats the argument of the pessimistic induction. There are various comments which should be made. Does it make sense to talk about things we can *never* come to know? This line of thought would drive us towards ontological structuralism. This of course is linked to the verificationist theory of meaning espoused by the old logical positivists. Statements that cannot be verified are simply meaningless. Of course any strict interpretation of such a principle would arguably render *every* statement in science, just as much as, for example, in theology, meaningless. We never know anything for certain except *perhaps* in logic or mathematics. So, if there are so many things we are not certain about, by the same token we personally may be quite happy to accept that there are things we are ineluctably ignorant about.

But is it true that mathematical structure really survives intact? In the most revolutionary episodes in modem physics, relativity theory and quantum mechanics, that is just not right. The new mathematics involves parameters like the velocity of light c in the case of relativity, or Planck's constant h in the case of quantum mechanics. It is only by letting c tend to infinity or h to zero that we recover something like the old mathematics of classical physics. But these limits are in general highly singular. A world in which h is actually zero is qualitatively quite different from a world in which h is different from zero, however small in magnitude it might be. To illustrate this consider squeezing a circle so as to try and turn it into a line. But a line just is *not* a very elongated circle – it has no inside and whether a curve is open or closed is an all - or - nothing matter. This is what mathematicians mean when they talk about singularities.

As another example, which is relevant to quantum mechanics, let us consider the limit of the classical wave equation of an elastic string for example, as the velocity of the waves tends to infinity. The character of the equation changes dramatically from what mathematicians call a hyperbolic equation to what they call a parabolic equation. Suppose the two ends of the string, of length L, are fixed, then the solution for the displacement y of the 'limit equation' is just $y = 0$. But for any finite velocity c, the solution of the original wave equation at an antinodal point is $y = \sin 2\pi\nu t$, where $\nu = c/2L$ for the fundamental mode of the string.

Consider the time average:

$$\bar{y} = 1/T \int_0^T \sin 2\pi\nu t \, dt$$

Then: $\bar{y} = 1/2\,\pi\,\nu\,T \bullet (1 - \cos 2\pi\nu T)$
$= L/\pi\, c\, T \bullet (1 - \cos \pi c\, T/L)$

For fixed T, however small, $\bar{y} \longrightarrow 0$ as $c \longrightarrow \infty$. But for fixed c, however large, we can always choose a T small enough to keep $\bar{y}$ *unequal* to zero. So the oscillatory behaviour of the string can always be revealed by averaging the motion over sufficiently short resolution times. So in structural terms, relativity and quantum mechanics genuinely involve new structure, not just the preservation of old structure. So is this not another example of a U-turn, like the abandonment of caloric or phlogiston? The best thing to do here is to say that the way mathematical structures 'develop' in physical theory has a certain natural, although not of course inevitable, aspect to it – natural, that is to say, to a mathematician.

There is of course a long tradition in natural philosophy that the physical world is constructed according to mathematical principles. This has a certain mystical appeal about it. For Plato, in the *Timaeus*, everything is constructed out of two sorts of triangle, a kind of mathematical atomism, and Galileo famously remarked that `the book of nature is written in the language of mathematics'. For the cosmologist James Jeans, God was a mathematician. So in this vein, in discovering the new mathematical structures are we learning to read the mind of God, as Stephen Hawking claimed in his famous best-seller *A Brief History of Time*.

Let us pursue this question of the role of mathematics in physics for a moment. There are two quite distinct cases to consider. In the first case mathematics provides a language to *represent* physical reality or at any rate some emasculated, idealised version of physical reality. We translate a physical problem into a mathematical problem and then, when we get the mathematical answer, just translate back into physics again. But in other cases we embed the physics in a wider mathematical framework, involving what can be called surplus structure, which *controls* the bit of mathematics actually used to represent the physical world itself. What do we mean by one bit of mathematics controlling another bit? In pure mathematics this is a familiar idea. Let us look at two simple examples.

To prove Desargue's theorem in plane projective geometry, the usual method is to introduce a point which does not lie on the plane, i.e. move to a three-dimensional geometry. In this setting we need only to assume the axioms of incidence to prove the theorem in the plane. If we restrict ourselves entirely to the plane we have to invoke a more powerful principle such as Pappus's theorem concerning properties of hexagons in the plane to get the proof. In a sense the third dimension is controlling, i.e. explaining, what is going on in the plane.

Or again consider the binomial expansion of the function $1/1\text{-}x^2$:

$$1 + x^2 + x^4 + \ldots$$

This only converges for $|x|<1$, and the reason is clearly related to the singular behaviour of the function at $x = \pm 1$. But what about the binomal expansion of $1/1+x^2$:

$$1 - x^2 + x^4 + \ldots$$

This function is perfectly well behaved for $x = \pm 1$, but the convergence properties of the series are now controlled (explained) by the singularity at $x = \pm \sqrt{-1}$, i.e. by the extension of the real line to the complex plane.

All this is familiar in pure mathematics. The surprising thing is that this sort of thing is also going on in modern theoretical physics. In particular in modern gauge theories of elementary particle interactions, the explanatory principles all operate in the realm of surplus structure! Let me quote from a well-known monograph by Henneaux and Teitelboim (1992, p. xxiii):

Physical theories of fundamental significance tend to be gauge theories. These are theories in which the physical system being dealt with is described by more variables than there are physically independent degrees of freedom. The physically significant degrees of freedom then re-emerge as being those invariant under a transformation connecting the variables (gauge transformation). Thus one introduces extra variables to make the description more transparent, and brings in at the same time a gauge symmetry to extract the physically relevant content.

It is a remarkable occurrence that the road to progress has invariably been toward enlarging the number of variables and introducing a more powerful symmetry rather than conversely aiming at reducing the number of variables and eliminating the symmetry.

Gauge theories are complicated by so-called ghost particles associated with these unphysical degrees of freedom. This is how the famous physicist Steven Weinberg (1996, p. 27) explains the role of ghost particles:

Each ghost field...represents something like a negative degree of freedom. These negative degrees of freedom are necessary because... we are really over-counting; the physical degrees of freedom are the components of [the gauge field] less the parameters needed...to describe a gauge transformation.

So ghosts (and indeed antighosts!) play a vital role in modern non-Abelian gauge theories. But these ghosts are not intended to have a real physical existence. They belong to the Unseen World in a more extreme sense than electrons or photons. One cannot but be reminded here of the famous Tibetan ghost traps that were supposed to ensnare the, to us non-existent, ghosts!

Tibetan Ghost Trap

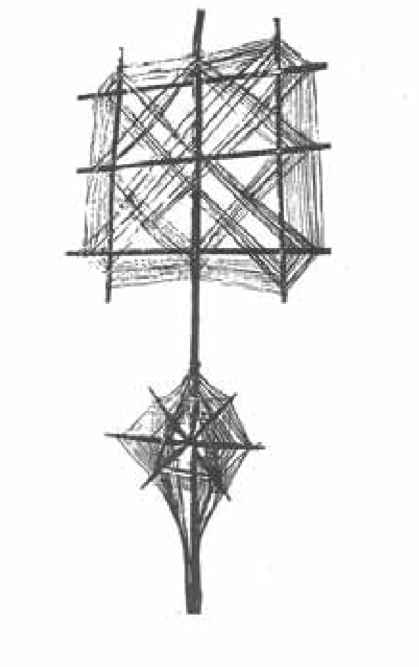

Fig 7

But what sort of world is the Unseen World? There is an ongoing theme in writing about science that behind and beyond the complex, variegated, diverse world of sensory experience there lies a simple, unified, integrated world that science is gradually revealing, that the Unseen World knits together the patchwork structure of the world of appearances, and provides the true account of the reality referred to in Plato's famous simile of the cave. As T. H. Huxley put it: 'The aim of science is to reduce the fundamental incomprehensibilities to the smallest possible number.' This theme of unification has generally been expressed by a scheme of reduction in which the sciences are arranged in a hierarchy, with sociology and psychology somewhere at the top, below that biology and then chemistry, the whole tower resting on the bedrock of physics. And physics itself is reduced to a unitary theory of everything, a TOE.

Such is the rhetoric particularly espoused by Nobel prize winners in physics applying for huge government grants to work on problems in fundamental physics. You might be forgiven for believing that the ultimate aim of science is to achieve a sort of one-off Humperdinck's Law from which everything else would be accounted for and explained.

But a strong reaction against this sort of wild talk has set in recently in philosophy of science. The pendulum has swung strongly in the opposite direction, promoting the disunity of science and the virtues of the Dappled World, the title of Nancy Cartwright's recent book.
The arguments here look at detailed case studies of what science is *really* like, and not just, in moments of wishful thinking, how we would *like* it to be. The description of real science provided by this work is much closer to the experience of the research worker at the cutting edge of the sciences than the sanitised account given in much of the popular science literature.

To be sure, warnings about the tendency of human beings to jump to conclusions about unification go back at least to the seventeenth century when Francis Bacon (1620/1960, p. 51) wrote:

The human understanding is of its own nature prone to suppose the existence of more order and regularity in the world than it finds. And though there may be many things in nature which are singular and unmatched, yet devises for them parallels and conjugates and relations which do not exist.

But has the pendulum swung too far?
We would like to explain our own point of view on this question. The idea of unification is essentially a regulative ideal. We may even want to *define* a concept of *scientific* rationality as one which invokes the simplest, most unified theory, to explain empirical phenomena. On this account religion, for example, is to be rejected, not because science shows it to be false, but because its acceptance would violate the canons of scientific rationality. This argument in defence of the scientific account is by itself clearly viciously circular. Its justification can, however, be provided in terms of the past record of scientific theories based on the pragmatic explanatory virtues of simplicity and unification, in producing successful novel predictions, the usual gold standard of scientific progress. So is it not rational to expect the same criteria to produce more successful science in the future? But such meta-inductions are always liable to fallibility. Perhaps at some deep level of explanation physics will get more complicated rather than increasingly simple. But that is why we talk of a regulative ideal. It does not have to be indefinitely achievable, but its past successes provide justification for pursuing the ideal as a leading principle of scientific investigation.

The difference between ourselves and Cartwright is essentially that she *likes* the Dappled World *à la* Gerard Manley Hopkins, whereas we want to get out our needle and thread and try to stitch the whole thing together.

So, let us try to summarise the status of the Unseen World. In philosophy there have always been two attitudes to the senses. The first is that the senses are linked not to reality, but to mere appearances. In the words of Parmenides' poem they access the Way of Seeming, not the Way of Truth. The senses are in effect a *barrier* interposed between us and reality. Reality can only be known, if at all, by reason or rational insight. The other view, a liberal and relaxed form of empiricism, is that the senses *link* us in an admittedly tenuous and fallible way with reality, and that science, in pursuing that link has at any rate in part revealed to us the Unseen World that lies behind and beyond the world of everyday experience.

We have said that religion makes no sense in the field of science. But what about the field of ethics. As we shall see the role of religion lies in its ethical concerns.

But in Chapter five we will show that religion may, on certain readings, give rise to scientific claims in the field of quantum mechanics! An alternative view of holism is also canvassed.

ENDNOTE

1. Michael Redhead in C. Cheyne & J. Worrall (eds), Rationality and Reality: Conversations with Alan Musgrave, 157 – 164, 2006, Dordrecht: Springer.

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CHAPTER 3 – RELIGION AND REASON

The word 'Religion' covers a wide spectrum. Of the major religions, from the Abrahamic religions through Buddhism, Hinduism and Sikhism to Confucianism and Taoism, each of these has, within it, considerable variety. Christianity moves from the complexity and splendour of the Orthodox Churches and Roman Catholicism to the simplicity and quietness of the Quakers. Islam ranges from the mysticism of the Sufi to the directness of the Wahabbi, Judaism from the orthodox Hassidic to the more relaxed Reform.

Buddhism in its earliest form (Theravada) is extant in Sri Lanka while changing at it moved though South East Asia (Mahayana) to China and thence to Japan (Zen) with its emphasis on the individual. A further centre of Buddhism (Tantric, Vajrayana or Mantrayana) developed a rich set of symbolism and practices of its own. Hinduism and Sikhism, which have only travelled recently, show less diversity. This is not to say that geography is the driving force behind such diversity but, in combination with time, it may help to explain it.

The three Abrahamic religions, perhaps, can be used to explain and illustrate the connection and relationship between religion and the host culture, and how both of these may change and influence each other with time, although this is not to say that they both change at the same pace over the short period. Judaism in its most recent history, following the *diaspora*, was ripped to a great extent from its geographical roots. It has been forced to exist in what have sometimes been hostile environments, and it is only within living memory that the link between the religion and the promised land of Israel – always an aspiration – has been restored.

As Judaism has migrated to new homes, it has become (with some dreadful exceptions) part of the fabric of its host societies, with members of that faith occupying settled positions at all levels of the host societies. However we must remember that Judaism is closely associated with Jewish ethnicity and is not a proselytising religion, thus its interest may be to adapt itself to, and live within, its surroundings rather than to change them more than is strictly necessary for this purpose.

Christianity in its shorter life has had a much more chequered history. It has always been an outward looking proselytising religion, and its first successful spread was eastwards. This spread was by sects (including Melkites, Nestorians and Jacobites) which were later considered heretical by the western churches, and in the early years of Christianity, the Middle East might have been considered the centre of Christendom. The spread westwards, via Constantinople and Rome, was slower and into quite different environments. Tragedy engulfed the church in the east and, although beleaguered remnants remain, it is largely forgotten now[1]. That said, these remnants may be seen as closer to the original early church[2]. Of the western church, with Orthodoxy migrating into Eastern Europe, Catholicism migrated into Western Europe and established itself as supreme over the long established Celtic church by the Council of Whitby. In this process, the Orthodox Church became part of each state in which it found itself (Bulgarian Orthodox, Serbian Orthodox, Russian Orthodox etc.) with each Metropolitan being an independent metropolitan of the national church.

The Catholic Church established itself as a powerful central authority, centred in Rome with the Pope claiming supremacy over the whole of Catholic Christendom. With the loss of eastern Christendom, the western churches were cut off from their heartland and the characters of the Orthodox and Catholic churches eventually diverged enough to allow the tragedy of the fourth Crusade. So we see that western Christianity adapted itself to existence in western feudal Europe, and by the later Middle Ages Christianity had become an essentially European religion. It was the Catholic Church which was the ancestor of the protestant churches of the Reformation and subsequent migration from Europe to the New World. It is interesting to compare the character of the modern churches with the character and power of the Catholic Church in the later middle ages and then to consider how the churches and their containing societies and cultures have changed since that time.

Islam is six hundred years younger than Christianity, remains firmly rooted in its heartland, has preserved the original language of its origins and informs many aspects of its adherents' everyday life. The existing history and development of Islam have their origin in societies and cultures very different from those in the west. To the extent that it has spread and consolidated across the Middle East in the wake of the Arab conquest and beyond, it can be called a proselytising religion. Consideration of the homes of Islam from the Middle Ages to the present day shows the close relationship between religion and every day society[3], although Turkey shows how Islam may exist in a secular state, notwithstanding pressure from some politicians for Islam to have a greater role. In its comparatively recent migration to the west, Islam has been carried by migrants who have brought their old culture with them (as is often the way with migrants) into very different host societies and cultures. That said, in Bosnia, where a secular European society had evolved with Muslims comprising about forty percent of the population, a more relaxed version of Islam seems to have obtained[4]. It is possible that in time Islam may similarly adapt itself to its newer homes in the West.

What do religions have in common? They have a core belief, or faith, and a set of ethical values, which follow from that belief, or faith. Perhaps this is an oversimplification, because some of the core beliefs are both complex and profound, while others appear to be relatively simple[5]. The tendency is to include a belief in God, or a pantheon of Gods, as essential for faith to be classified as religion, but it is fairly well admitted that Buddhism, without a belief in any god, is a religion[6]. Indeed it has been suggested (by Tony Benn, when considering those religions to be included in the curriculum for Religious Education) that Communism should be included as a religion; after all it does have some core beliefs and ethical values, whether or not one may agree with them. Similarly one might consider Atheism as propounded by its zealous advocates as a religion; although in this case it might be said that the ethical values propounded by some of these are imported from the religion of their upbringing, or that of their original teachers. Although it may be said that these ethical values are underpinned by reason and humanity, it must be remembered that the ancient classical philosophers were only made accessible, after they were lost in the dark ages, by Arabic Christian and Islamic scholars and subsequently propagated to the West by Christian translators. Thus any subsequent discussion and debate must have been informed to some extent by the faiths of these men and their successors down to modern times.

Dawkins, and Dennett in more philosophical detail, take the view that knowledge is confined to that which is 'observable'. [But see Chapter 2]. This is the province of science, and to say this is not in any way to disparage science. However, this calls into question the view of the observer, the technique of the observation and what is actually being observed. When we speak of science, it is perhaps important to remember that we are just speaking of the current state of a constantly evolving and expanding body of knowledge and understanding. Today's certainty is tomorrow's past mistake, and unless we can be certain that we have reached the stage such that there are no further discoveries or enhancements of understanding possible, we must acknowledge that uncertainty cannot be eliminated entirely. Although, as each advance is made, hindsight will say that this was, of course, obvious. As we shall see, even scientific or mathematical knowledge may outrun the power of observation or proof, and it is important to remember that failure to prove something is positive is not a proof that it is negative. The ethical values, which follow from religion, may be considered as based upon knowledge outside of that which is observable. Does this mean that such ethical values, underpinned by the moral certainty of religion, are to be discounted? We suggest that we have two types of knowledge: the laws of science[7], and the laws of morality underpinned by religion.

Religion offers us, in the sense of reason, a *guide* to the moral law, but only a guide. We can accept the moral law as our best intention to meet the demands of religion. That is to say that we accept the rôle of religion as a tentative guide to believing in the moral law. This implies some element of choice, both in the religion, which we espouse, and the rules of conduct, which that religion enjoins. Thus each religions package need not be taken on an all or nothing basis, and each religion has something different to offer. This is not to say that we can mix and match *ad infinitum*, but we can select a religion and those elements within it, which provide a reasonable and consistent framework within which to live. This may be influenced – for example – by upbringing, a charismatic teacher, the beauty of the buildings or ritual, or the behaviour and characteristics of the adherents of a particular faith.

It should be remembered that in historical times zealous adherents, or leaders, of most religions have performed acts, which would be deplored today and that these acts live on in actual or (powerful) folk memory. On the other hand, the world has reason to be very grateful for the good, which has been effected in the name of those same religions. Sadly the bad is more easily remembered and the good is more easily forgotten.

Reason applies the rational power of the mind as compared with the 'passions'[8]. The latter are like Plato's two horses, one black and one white, which are controlled by a charioteer who is akin to the rational mind. Reason may take us beyond the evidence as in Peircean *abduction*. When we start to think about evidence for faith, we raise very difficult questions. The historical evidence is far back in time, and in many cases overlaid by commentary upon commentary; part of what survives in written form conflicts with modern scientific scholarship. The resulting theology, at first sight, often has contradiction and stunning complexity.

According to St Thomas Aquinas, still widely taught in modern Catholic teaching, for example[9], grace enables one to go beyond reason to explore the mysteries of faith, but reason is a measure of our limited ability to think things through. Reason delivers, perhaps, the *probability* of the argument. By its very definition probability implies uncertainty, although mathematicians and engineers apply formulae to move towards overcoming this uncertainty and increasing objectivity. An interesting example of this is the clear exposition and rigorous application of the technique by Richard Swinburne[10].

So how does reason work? There are many areas where faith impinges on the world of science, and that is where belief in science acts back in relation to faith-centred belief. In other words the role of doctrine is not immune to the demands of reason. This is the rôle of Vatican II's demand on how faith can be altered to agree with the facts of science.

This is the view of modern Catholic teaching. But, of course there are many other views. David Hume, for example, argues that 'reason has become the slave of the passions', while Karl Barth has advocated a view of God as characterised by faith alone, a view widely acknowledged by evangelical fundamentalists who see the word of God as revealed in unalterable terms through the Biblical record. Immanuel Kant has argued for the Categorical Imperative, and makes a rational claim for this, which can be disputed, and so on. Muslims take the Koran to be the direct revelation of the Word of God by the Angel Gabriel to the Prophet Mohammed, and have preserved it in the original language of that revelation.

One author's view [MR], as a liberal Catholic, is as follows. We have our faith centred on our reason. But what makes a faith? It extends out beyond reason but there are many varieties of faith that fit with reason. We want to argue that our view of faith involves a system of cultural, historical, and other textual knowledge that makes sense to us, but of course, we must recognise that other faiths are available. So we adopt our own faith, but admit that other faiths make equal sense to other people who adopt them. (Indeed, the out-and-out atheist must be allowed, as discussed previously, although the more comprehensive view of religion justifies the rôle of explanation in moral discourse). Thus we can accept our personal faith, but freely accept that we have no certainty that we can know the supreme reality that rules everything.

Another view [SG] is that religion can be thought of as having a belief together with a code of conduct enjoined upon its followers. On the one hand there is the belief in the theology, with all that that implies, on the other hand there is the expected behaviour of the religion's adherents. So if we consider any religion, we can start by examining the theology or the core beliefs and their implications; this might be considered as a top-down approach. Alternatively, we can start by examining the actual behaviour enjoined upon the followers together with the implications of such behaviour; this might be considered a bottom-up approach.

Examination of the philosophy of any religion may be difficult and open to dispute, calling into question its actual purpose, while the actual teachings of how its adherents should behave with regard to one another are often simpler. The first approach is more difficult but may lead to a more profound insight and understanding, while the second approach is more accessible and may lead to some insight. For example, we can ask if the teachings of Jesus Christ are to be given weight *only* because of *who* He was, or if they may be considered to have weight on their own merits. [This is the essence of Plato's Euthyphro conundrum]. The parable of the Samaritan would suggest that conduct and common humanity are, at least as important, if not more important, than specific belief; so it may be possible to practice love for one's neighbour, forgiveness etc. without being too concerned with the complexities of the theology. We can see that practising this code (forgiveness and helping one's neighbours) gives benefit to humanity by, amongst other things, reducing violence and relieving suffering. Thus, we can follow the code of conduct because we believe it is right to do so[11], and neither in expectation of reward nor in fear of punishment, although both these possibilities are raised.

Our own view, then, is that moral judgements are not made up by us but are mandated by the idea of God. In this sense our moral judgements are involved with a higher unifying principle and this ties in with the role of unifying principles in science itself.[12]

Gödel has argued that some truths of arithmetic are not provable. Dennett has claimed that evolution involves 'cranes' [algorithmic processes] rather than 'skyhooks' [miraculous events] which he takes Gödel as arguing for. He is against the view that truth versus provability makes sense. We present a new version of Gödel's argument that shows that Dennett is arguably wrong here.

ENDNOTES

1. Philip Jenkins, 2008, *The Lost History of Christianity*, New York: Harper Collins. See also Diarmaid MacCullock, *A History of Christianity: The First Three Thousand Years*, 2004, london: Allen Lane.

2. William Dalrymple, 1998, *From the Holy Mountain*, London: Flamingo.

3. Albert Hourani, 2005, *A History of the Arab Peoples*, London: Faber and Faber. This is also well illustrated by the medieval traveller Ibn Battuta and the modern traveller Tim Mackintosh-Smith.

4. Noel Malcolm, 1994, *Bosnia A Short History*, London: Papermac.

5. For an overview of problems in the philosophy of religion see, for example, Ronald W. Hepburn (1988) ' The Philosophy of Religion' in G. H. R. Parkinson (ed) *An Encyclopaedia of Philosophy*, London: Routledge, pp. 857-877.

6. The case of Hinduism is interesting. It is more a way of life without congregations going to church, or officials saying what to believe. Indeed priests may tell the uneducated what to believe, sometimes to suit their own purposes, but there is no authority to contradict them.

7. For views on the nature of scientific laws see Marc Lange (2008) 'Laws of Nature', in S. Psillos and M. Curd (eds) *The Routledge Companion to Philosophy of Science*, Abingdon: Routledge , pp.203-212. We don't subscribe to miracles, subverting the laws of nature. God may work through the laws of nature, providing a framework for His ultimate rule. Thus, our rejection of Professor Hawking's atheistic espousal of so-called M-theory. See Stephen Hawking and Leonard Mlodinow, *The Grand Design: New Answers to the Ultimate Questions of Life* [Transworld Publishers: Bantum Press, 2010]. Hawking subscribes to the Anthropic Principle that sees the role of fundamental constants [and the mutiverse or set of universes} as allowing 'fine-tuning', and raises again his idea of a 'no-boundary condition' in which the rôle of the time-coordinate becomes, in the limit, a feature of a fourth spatial-coordinate. This is speculative, but again may involve a strong theological component as evidence of God's involvement of fundamental physics and the development of human consciousness.

8. This simplistic view has been challenged by philosophers such as MacIntyre and Williams, who seek to merge rationality and the passions, arguing that from a richer description of rationality, this necessarily involves the emotions.

9. R. McBrien, 1994, *Catholicism*, New York: Harper Collins, p.15. In this magisterial work on modern Catholicism rejects rationalism (the belief that reason alone could grasp the mysteries of faith) but also fideism (the belief that an uncritical faith, apart from reason) is sufficient to grasp God's revelation.

10. Richard Swinburne, 2007, *Revelation From Metaphor to Analogy,* Oxford: Oxford University Press, pp. 345-356.

11. Interestingly a programme on Channel Four, which asked the public to rewrite the Ten Commandments, resulted in the most important of the new commandments to be to treat others as one would wish them to treat oneself. This is not unlike, 'Love thy neighbour as thyself', and maybe shows the universal nature of such teaching.

12. The alternative view that human beings are just glorified form of apes and other lower creatures is quite incredible. It does not allow for any sense of right or wrong, and leaves out the idea of human beings as special creatures, with immortal souls, as most religions demand, under the rule of God. By 'immortal souls' we mean really our acknowledgement of God's role in moral discourse. We agree that early human beings may have a rudimentary 'soul' that came to fruition in *homo sapiens.* See, for example, Keith Ward, 'Defending the Soul', Hodder and Stoughton, 1992, pp.64.

CHAPTER 4 – GÖDEL'S ARGUMENT

Isn't it a bad thing to be deceived about the truth, and a good thing to know what the truth is? (Plato, *The Republic*)

In Chapter 15 entitled 'The Emperor's New Mind, and Other Fables' (see D. C. Dennett, *Darwin's Dangerous Idea :Evolution and the Meanings of Life*, New York: Simon & Schuster, 1995) Dennett argues in his racy style as follows:

'The attempts over the years to use Gödel's Theorem to prove something important about the nature of the human mind have an elusive atmosphere of romance. There is something strangely thrilling about the properties of 'using science' to such an effect. I think I can put my finger on it. The key text is not the Hans Christian Andersen tale about the Emperor's New Clothes, but the Arthurian romance of the sword in the stone. Somebody (our hero, of course) has a special, perhaps even magical, property which is quite invisible under most circumstances, but which can be made to reveal itself quite unmistakably in special circumstances: if you can pull the sword from the stone, you have the property; if you can't, you don't. This is a feat or a failure that everyone can see; it doesn't require a special interpretation or special pleading on one's own behalf. Pull out the sword and you win, hands down. What Gödel's Theorem promises the romanticaly inclined is a similarly dramatic proof of the specialness of the human mind. Gödel's Theorem defines a deed, it seems, that a genuine human mind can perform but that no impostor, it seems, no mere algorithmic-controlled robot, could perform.'

Let us begin with a bold assertion. Ascertaining the truth in any field of enquiry is intrinsically good. Notice it is not denied that actions which employ or implement our knowledge may be morally bad. One only has to think of nuclear physics in the context of weapons of mass destruction. Nor is it denied that the truth about some impending calamity, for example, may lead to mental anguish which has to be balanced against the opportunity to display courage and fortitude Nor is it claimed that telling the truth to other people is always the right thing to do since one may have good reason to believe that they will employ the knowledge in a harmful way. Nor is the fact/value distinction denied, that is a separate issue.

This claim is of course controversial: it advocates knowledge for its own sake. Many people might argue that *interesting* knowledge is good, not just knowledge *per se*. The truth about the exact number of hairs on one's head might seem totally uninteresting but from the point of view of arguments about nature versus nurture might prove not to be so! So `interesting' is something of a weasel word that is hard to pin down in any absolute sense.

But if it is granted that truth is a good thing, then reliable methods of ascertaining the truth will also be a good thing. In science, notoriously, *certifiable* truth is hard to come by. In other fields, such as science and theology, claims to truth usually appeal ultimately to accepting some version of simplicity, or some form of inspirational revelation, the authenticity of which can always be challenged, and in ethics, aesthetics, politics, economics, the law courts and the social sciences generally, truth claims seem equally problematic.

So let us turn to mathematics as a field in which the possibility at least of genuinely certifiable truth seems more promising.

At first glance, truth in mathematics seems to equate with proving theorems. But we need to be careful here. There are basically two sorts of mathematics. An example of the first sort is group theory, where one is really concerned with `unpacking' the logical consequences 'locked up' so to speak in the axioms of group theory. The truths of group theory are analytic in the sense that the conditional statement 'If the axioms are true then the theorems are true' is itself a logical truth. One is not asking the question: are the theorems true *per se*? Rather one is claiming, if the axioms are true under some interpretation, then the theorems are also true under that interpretation.

Contrast the situation in group theory with that in number theory. Here the hope is that the axioms are true of an *intended* interpretation, i.e. the natural numbers 0, 1, 2... Of course we know from the work of Gödel and Tarski, that it is not possible to prove or even, in a certain technical sense, to define, all the arithmetical truths. So truth outruns provability (or definability). But does this just mean that arithmetical truths outrun *knowability*? The curious thing is that this does not follow.

Gödel, for example, famously argued that there are true sentences of arithmetic (true that is in the intended argument interpretation) which cannot be proved from the Peano axioms[1], and yet we can claim to know them to be true. This has been taken up by John Lucas and more recently by Roger Penrose to argue that minds can do things which computers cannot do, and hence that minds cannot be (digital) machines.[2] Of course we can always go to a stronger system of axioms to prove the Gödelian sentence, but then we can re-Gödelize, and so on, ad infinitum. Put succinctly,

For any proposed Gödelian sentence there exists a machine which can deal with it (i.e. prove it) (1)

but

It is not the case that there is a machine which can deal with all Gödelizations (2)

Essentially Lucas and Penrose are invoking (2) to resist the claim that minds are machines, whereas many people argue that (1) is all that is needed to establish exactly the reverse.[3]

But how do we know the Gödelian sentence G (which essentially announces its own unprovability in a suitable scheme of translation) really does express a truth about the natural numbers. There are various answers that can be given. The simplest just says that if G is false it would be provable and hence true, whence by *reductio* it cannot be false. But this presupposes that the axioms of Peano arithmetic are true. So how do we go about convincing our sceptical opponent that that is the case?

Or again, it is often argued that one can prove in first-order Peano arithmetic the conditional statement

$\mathrm{CON} \rightarrow \mathrm{G}$ (3)

where 'CON' expresses the consistency of the Peano axioms. Now we cannot prove CON, since if we could we could prove G, which by

Gödel's first incompleteness theorem we cannot do (this is just the second incompleteness theorem). But if Peano arithmetic is sound then (3) is true, and we can argue *inductively* for the truth of CON (no-one has so far discovered a contradiction in Peano arithmetic). But combining this inductive argument for the truth of CON with the truth of (3), we can now detach the consequent in (3) and infer the truth of G. But this argument again presupposes the soundness of Peano arithmetic and in particular the truth of its axioms.

We should now consider this problem of how we know the truth of the axioms and in so doing show how the Lucas-Penrose argument can be recast in a much simpler and more transparent form.

The Peano axioms are of three sorts. First there are axioms of the form:

1. Zero is a number.
2. Every number has a unique successor.
3. If two numbers have equal successors then the numbers are equal.
4. Every number other than zero is the successor of some number.

These axioms are arguably analytic in the sense that they express defining properties of the numbers 0, 1, 2, 3.... If any of these axioms were false we would not be talking about numbers.

But then there is the notorious fifth axiom, the induction axiom (more accurately an axiom schema in first-order logic). This says that for any admissible predicate[4] F, if F is true of zero and if, given that F is true of n then it is true of the successor of n, then it is true for all n.

Poincaré ([1952]) famously claimed that this was not an analytic truth, but forced itself on us with such conviction, that it was a candidate for the elusive synthetic *a priori.* The induction axiom certainly cannot be proved from the first four axioms, so what is the source of the conviction that Poincare talks of? We shall return to this in a moment.

The remaining axioms of Peano arithmetic introduce recursive definitions of addition and multiplication, essentially defining addition in terms of repeated application of the successor operation, and multiplication as repeated addition. These axioms can also be arguably regarded as analytic.

To appreciate the significance of the more mysterious fifth axiom, it is useful to consider a weaker form of arithmetic that Lucas calls sorites arithmetic which employs all the axioms of Peano *except* the induction axiom. A carefully formulated version of sorites arithmetic is called (following Raphael Robinson) the system Q in the standard text-book of Boolos and Jeffrey (1980). Sorites arithmetic essentially allows us to move from one number to the next and repeat the operation a finite number of times. So we can formulate the usual procedures of, for example, long multiplication and verify statements like 3 x 6 = 6 x 3 or 279 x 631 = 631 x 279. Indeed we can check out the commutative property of multiplication for any pair of numbers m and n. But the proof gets longer and longer as m and n get bigger and bigger.

So, in sorites arithmetic

For all pairs (m, n) there exists a proof that m x n = n x m (4)

But what we cannot do is to switch the order of the universal and existential quantifiers to get

There exists a proof that for all pairs (m, n) m x n = n x m (5)

(5) expresses the proof of the commutative law of multiplication in arithmetic.

But for (5) we need the induction axiom. Why is this? Because the proof referred to in (4) depends on the pair of numbers chosen, and while always remaining finite in length, increases in length without limit as the numbers get bigger and bigger. So there is no finite proof that will work for all pairs of numbers, which is what we need for (5).

But we can argue that

For all pairs (m, n), m x n = n x m (6)

is nevertheless true in sorites arithmetic.

We have argued that the axioms and hence the theorems of sorites arithmetic are analytically true, so we can replace (4) by

For all pairs (m, n) it is true that m x n = n x m (7)

But (7) is strictly equivalent to

It is true that for all pairs (m, n), m x n = n x m (8)

But (8) just says that the commutative law of multiplication is true.

Effectively, truth commutes with the universal quantifier, whereas provability does not! So we have here a case in which certifiable truth outruns provability.[5]

The same argument applies to many other theorems that can not be *proved* in sorites arithmetic, such as the commutative law of addition or the associative laws of addition and multiplication, and, more particularly it applies to establishing the truth as against the provability of the induction axiom itself.[6] So here is a simple argument that human minds can know the truth of statements which can be expressed or represented in a system but cannot be proved in the system, but without employing the complexities of understanding the proof of Gödel's theorem! Of course we can go on to demonstrate that every consistent, axiomatizable, extension of sorites arithmetic is incomplete in the sense that there are true sentences of arithmetic which cannot be proved in any such extension. This requires the much more complex Gödelian arguments.[7]

We have only taken the first step of showing that sorites arithmetic itself is incomplete. But the important ingredient is to see how truth can outrun provability in the interesting sense that this is not the case just for analytic truth, but for truths like the induction axiom, which are in no clear sense constitutive of the *meaning of number*.[8] Rather our argument depends on having a concept of truth[9].

Notice we are not saying that a view of religion can be captured in, say, an axiomatic form. Gödel's Theorem is true of Peano's arithmetic. But what it shows in essence is that human thought is ineffable, and this allows room for a religious view.

It has been argued by Dennett [1995] that human minds and their operation are analogous to computers running programs – or rather, carrying out algorithms. This brings out the question of whether machines can think and – by applying the Turing test – whether man and machine can be distinguished, and how (and by whom or what) such distinction would be measured. It may be admitted that computers are able to *simulate* the performance of the human mind, and this simulation may be very effective, such as to deceive many people. For example various chess playing algorithms may beat most players, but the chess player will not carry out all the exhaustive analysis carried out by the algorithm, instead the player will play using – in part – intuition. But it has yet to be shown that the computer or machine is capable of the '*eureka moment*', that leap of intuition or instinct which defies analysis. Our own point is to look at a *singular* example of the fact that truth outruns proof. We claim that this throws additional light on what Gödel's argument is all about.

But can it now be concluded that minds are not machines? If you are moved by constructivist considerations, you may want to deny that the truth of statements involving universal quantification over infinite domains has any clear meaning. But, for a Platonist, and that means most working mathematicians, the argument may seem compelling.

But now we have come full circle. If you accept you are not a machine, then this at the very least opens the door to the possibility of how you can *understand* the evaluative claim we started with (even if you don't agree with it), that ascertaining the truth is intrinsically good, something that no machine presumably could do!

We move next to consider the role of quantum mechanics and the nonlocality issue to allow room for Divine Action.

ENDNOTES

1. Gödel assumed the Peano axioms to be ω-consistent. This was later relaxed by Rosser to the weaker assumption of consistency. See Machover (1996). For an up-to-date version of Gödel, along the lines of Boolos and Jeffrey, see Peter Smith, 2007, 'An Introduction to Gödel's Theorems', Cambridge: Cambridge University Press.

2. See Lucas (1961), (1970) and Penrose (1989), (1994). Penrose employs a simplified form of the Gödel argument due to Turing.

3. For complete references to the many different types of response to the Lucas-Penrose argument, see Lucas (2000) and Penrose (1994).

4. What predicates are admissible depends on whether we choose to employ first- or second-order logic.

5. In more formal terms, we have sought to demonstrate that sorites arithmetic is ω-complete in a semantic sense, while recognizing that it is ω-incomplete in syntactic

terms. From this perspective, Gödel's theorem establishes the ω-incompleteness of Peano arithmetic. The truth of the Gödel sentence then follows from an argument similar to the one given in the text showing that Peano arithmetic is ω-complete in a semantic sense. Trivially, the fact that truth outruns provability follows from bivalence for any formal system which is incomplete. But of course we are interested in knowing which of `p' and 'not p' is true.

6. It is important to notice that none of these theorems can be proved using the logical principle of Universal Generalization (UG). UG allows us to pass from the fact that a typical member of a collection possesses a property to the claim that all members of the collection possess that property. But the decisive point is that, for the purposes of these theorems, all the numbers are uniquely different – there is no such thing as a typical number. We stress again that we are talking of theorems which are true of the intended interpretation. There will, of course, be non-standard interpretations of sorites arithmetic (in first-order logic), but these will include an initial segment isomorphic to the natural numbers. It is with regard to this initial segment that we are claiming the theorems to be true. We owe this point to Richard Healey. An additional remark: these theorems could be proved if we employed an infinitary logic incorporating the so-called ω-*rule*. But such logics cannot of course be implemented on a machine.

7. See, for example, Boolos and Jeffrey (1980). Notice that whereas the syntactic proof can be formalized, the semantic proof cannot. This is because, in order to formalize it, we would have to express the notion of truth. But this is ruled out, as we have seen, by Tarski's theorem on the undefinability of truth, whose proof is closely connected to Gödel's theorem. The definable truths are at most denumerable, whereas there are nondenumerable sets of numbers. See Berto (2009) p.154, following Smullyan (1992) p.112. Notice that the Gödel theorems say that if a sufficiently strong version of arithmetic is consistent then the result follows. But we cannot prove consistency (by the second theorem). This is the basic conundrum. We have to assume consistency, but this moves us outside the formalized theorems. Inductive support, as we have seen, is provided by the fact that no inconsistencies have so far been produced. Gödel makes great stress on the constructivist aspect of syntax, but the argument leads to a conundrum, as we have seen, which can only be solved in terms of the semantic approach in the way we have described.

8. Sorites arithmetic is the weakest system that allows this possibility; i.e. for still weaker systems, truth outruns provability only for analytic statements.

9. This has been challenged by Raatikainen (2005) who claims correctly that Tarskian semantics invoke a failure of showing that truth outruns provability. But we do not of course have to believe in Tarskian semantics. See, in this respect the views of R L Kirkham, 1997, *Theories of Truth: A Critical Introduction*, Cambridge Mass: MIT Press. Further points are made in the reply by Lucas and Redhead (2007).

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CHAPTER 5 – QUANTUM MECHANICS - THE TANGLED STORY OF NONLOCALITY[1]

1 Introduction

There are many purported proofs of nonlocality in quantum mechanics. A representative selection of such proofs will be critically examined so as to expose as precisely as possible the assumptions on which they are based. In order to keep the discussion to a reasonable compass the detailed treatment will be restricted to phenomena involving so-called entangled states, i.e. many-particle states that cannot be expressed as a simple product of one-particle states, but only as a superposition of such products.

This chapter is predicated on the assumption that nonlocality in the sense of instantaneous causal action-at-a-distance is to be avoided if at all possible, on pain of violating at any rate the spirit of special relativity. It is, of course, controversial as to what exactly special relativity does prohibit. The view is taken that relativity is more than just a phenomenological invariance principle, and is essentially grounded in the causal structure of spacetime[2] Thus there are essentially three principles at issue in discussing possible conflicts with special relativity.

Firstly, there is the First Signal Principle (FSP), which asserts that we cannot transmit information in the form of a signal faster than the speed of light in vacuo. Then there is the Invariance Principle (IP), which claims the invariance of physical theories under Lorentz transformations, i.e. if any physical process is allowed then so is any Lorentz-boost of that process. Finally, there is what we will call the Philosophically Grounded Invariance Principle (PIP), which asserts that causal influences cannot operate outside the light-cone and thus provides a causal underpinning for the characteristic light-cone structure of Minkowski spacetime.

Note the following logical relationships between these principles:

$$\text{IP} \not\rightarrow \text{FSP} \qquad (1)$$

and

$$\text{PIP} \rightarrow \text{FSP}, \qquad (2)$$

but

$$\sim\text{PIP} \not\rightarrow \sim\text{FSP}. \qquad (3)$$

In our view PIP is the basis of special relativity. ~PIP means that causal influences operate at spacelike separation, i.e. propagate faster than light, but in view of (3) this is perfectly consistent with the FSP, i.e. no signal can be sent faster than light. The reason for this is that to constitute a signal the causal process must be suitably controllable, and this may be inherently impossible in the quantum situation. Some authors such as David Bohm have advocated interpretations of quantum mechanics that do exactly that, namely, allow superluminal causal processes but not superluminal signalling. If special relativity entails FSP but not PIP, then Bohm works. But if – as we believe – special relativity entails the stronger PIP as well, then Bohm is definitely inconsistent with special relativity.

The Bohm interpretation is also deterministic. It can therefore be objected to on two counts:

1. the *scientific* count of fudging the relativity issue;
2. the *theological* count of not allowing 'room' for divine action.

Point 2 seems prima facie right, but is of course disputed by philosophers and theologians defending a so-called compatibilist position between determinism and the possibility of Divine Action, free will, etc. For a vigorous defence of incompatibilism see the work of John Lucas[3].

We shall therefore be looking particularly at indeterministic settings and examining the question of whether proofs of objectionable forms of nonlocality can be blocked, thus remedying the two defects in Bohm-type interpretations noted above (despite their undoubted attraction in terms of picturability – trajectories – and the absence of a measurement problem). We also examine recent demonstrations of nonlocality in the context of relativistic quantum mechanics. We conclude that an indeterministic framework in which measurements actualize potentialities (as advocated by Abner Shimony, for example) offers the best prospect for avoiding the twin objections to the Bohm approach.[4]

2 The EPR and Bell Arguments

It is often maintained that quantum mechanics is obviously 'nonlocal,' in that it treats of extended wavefunctions, so an electron in a hydrogen atom, for example, is in some sense everywhere at once.[5] But things are not that simple. There are three issues involved: (a) Are electrons 'spread out'? (b) If they are, do they nevertheless have local properties, such as propensities to be at particular locations, the 'spreadingoutedness' reflecting the fact that a whole spectrum of such properties can be possessed by the electron at the same time? (c) Can these local properties 'act' on one another instantaneously at a spatial separation (speaking nonrelativistically for the time being)?

The puzzle starts with (a). When you measure the position of an electron you find it somewhere, so it is tempting to think that the sense of 'nonlocal' in (a) reflects an ignorance of where the electron is, rather than an 'ontological' spreading out. But if you answer 'yes' to (a) then (b) also gets a 'yes,' so the properties themselves are not 'spread out.' (c) trivially gets the answer 'no' if you answer 'no' to (a), but 'yes' if you go along with 'yes' to (a) and (b). This is really the famous issue of the projection postulate. If we change the propensity 'here' to one, the propensity everywhere else goes to zero, and that looks like action-at-a-distance.

Then, what about the famous two-slit experiment? If the answer is 'yes' to (a), there does not seem to be any extra nonlocality involved in the effect of opening and shutting the second slit. If the answer is 'no' to (a) then it is not clear whether any instantaneous effects can be propagated – it depends on the detailed 'hidden-variable' dynamics of the experiment.

So, it is all very confusing. Clarification began in 1935 with the Einstein. Podolsky-Rosen (EPR) argument[6] for the incompleteness of the quantum formalism The EPR argument deals with the quantum mechanics of *two-body* systems. This opens up the possibility of a much more precise discussion, but also new sorts of issue arise: (A) If you have two distinguishable particles in quantum mechanics, like an electron and a proton, does each of them have individual properties?[7] If the answer to that is 'yes' then we can go on to ask the question: (B) Can these properties be affected nonlocally by events occurring even at spacelike separation. EPR assumed the answer to (A) was 'yes' and proceeded to pose a dilemma: the formalism of quantum mechanics implies either that:

(B) gets answered 'yes'

or

(a) gets answered 'no.'

EPR assumed (B) is answered 'no,' and hence argued that (a) gets the answer 'no.'

The crunch came in 1964 when John Bell showed[8] that, if the answer to (a) is 'no,' then the answer to (B) is 'yes'! So the conclusion of the two arguments put together is: the answer to (B) is 'yes.' But since the argument is predicated on assuming that the answer to (A) is 'yes,' the conclusion should really be rendered as: the answer to (B) is 'yes,' *or* the answer to (A) is 'no.' Here, the first disjunct (B) offers us action-at-a-distance between individual properties. The second disjunct (A) offers us 'nonseparability,' a holistic involvement of the two particles, in which not all their properties can be attached individually to each particle. But unfortunately that is not the end of the story, because Bell's argument brought in some other assumptions, call them collectively P, so the conclusion is really: either P is false, or we have nonlocality (in the precise sense that we have either action-at-a-distance or nonseparability). So if we want to avoid the unpalatable conclusion of nonlocality then we can always escape by denying P. So we had better delve into P. We must be a *little* more precise about the details of Bell's argument.

3 Presuppositions of Bell's Argument

We have introduced (a) in the version: Are electrons spread out? But the question is much more general. Do superposed states 'spread' properties generally among the properties appropriate to the component states? So far we have talked about position, but for a general observable *a*, in a superposition of eigenstates of the associated operator *A*, what can be said about the value of *a*? The original EPR argument did apply to the positions of two particles and purported to show that in an appropriately chosen superposed state, the positions of the particles were sharp, i.e. not in any sense spread out. But in 1951 Bohm[9] developed a version of the EPR argument that used the spin components of two spin-1/2 particles projected along a particular direction, say the Z-direction, in a state, the so-called singlet state, involving a superposition of products of eigenstates of the spin-projections along the Z-axis, and demonstrated that, even in such a superposition, the spin-projections of each particle had a definite sharp value.

Bell employed the Bohm version of the EPR argument, but now considered spin-projections for the two particles along *different* directions. Assuming that correlations between spin-projections predicted by quantum mechanics could be expressed as a weighted average over the products of the spin-projections for given values of the hidden variables and the values of the spin-projections were determined by the 'hidden' variables describing the 'complete' state of the source, he derived an inequality, the so-called Bell inequality between correlation coefficients, which was violated by

the predictions of quantum mechanics, and also, as it turned out, by experiment.

So one can now identify two elements in the additional assumptions P, used by Bell in his original paper to arrive at his conclusion.[10] First, a probability structure was laid out, exhibiting the values of the spin-projections as random variables over a classical probability space equipped with a probability measure expressing our ignorance as to the exact hidden state of the source. Secondly there was an assumption of determinism. In particular, the first assumption identified a joint probability distribution for the values of spin-projections along different directions for the same particle although, according to the usual interpretation of the uncertainty principle, such components for the same particle could never be measured simultaneously. We shall call this particular assumption JD (for joint distribution).

Two major controversies in the literature have arisen out of Bell's argument:

1. Can one get the argument off the ground if one denies JD?

2. Can one get the argument off the ground if one denies determinism?

With regard to JD, things looked promising when a proof of the Bell inequality was developed by Henry Stapp and Phillipe Eberhard,[11] which did not use the machinery of formal probability spaces at all, but expressed the relevant correlations directly in terms of limiting frequencies in a long run of repetitions of the experiment. Making appropriate randomness assumptions about these sequences, it appeared that one could derive the Bell inequality without committing to JD.[12]

With regard to the second controversy concerning determinism, it is argued against Stapp that determinism was a concealed assumption in the Stapp-Eberhard proof. The issue here related to the necessity of using counterfactual formulations in the Stapp-Eberhard proof. This turned on the question, what can one say about the possible result of a measurement that one could have performed but did not? Could these counterfactual results for one particle vary according to what measurement was carried out on the remote particle? If the measurement results were generated indeterministically, it is claimed, such variation could be routinely expected in the absence of any mysterious nonlocal influence of measurement procedures on one particle affecting the state of the remote particle.[13]

So, can one prove the Bell inequality assuming indeterminism? The answer is 'yes,' and such a proof was explicitly produced by John Clauser and Michael Horne[14] (who developed a general setting for ideas presented more sketchily by John Bell).[15]

This proof involves giving up the idea that the measurement results are determined by the hidden variables, but allows these results to be linked stochastically to the hidden variables describing the complete state of the source and/or the measuring apparatus. Since these measurement results are no longer to be thought of as random variables on a single probability space, there is no commitment to JD (further reinforcing the argument of Svetlichny, Redhead, Brown and Butterfield).[16] But in this proof a new formulation of locality has to be provided, including a potentially controversial assumption of conditional stochastic independence of measurement outcomes for the two particles – conditional, that is, on the precise specification of the hidden variables.[17]

4 The Algebraic Proofs of Nonlocality

The proofs of nonlocality discussed so far have all depended on proving a Bell inequality relating certain statistical correlations and then using the fact that this inequality is violated experimentally, and also by the predictions of quantum mechanics. A quite different approach has tried to demonstrate that local hidden-variable theories are actually self-contradictory, so we would not need to do experiments, involving inevitable interpretation, with auxiliary assumptions about the working of the measuring apparatus that can always be called in question, to avoid impugning locality. This approach is generally referred to as the algebraic approach to demonstrating nonlocality, since the contradiction demonstrated amounts to showing that two unequal numbers, such as zero and one, are equal.

An algebraic contradiction for the hidden-variable description of a single system described by a state-space of dimension greater than two had been demonstrated by Simon Kochen and Ernst Specker in 1967,[18] assuming that value assignments to physical magnitudes obeyed the same functional relationships as the associated quantum-mechanical operators. The resolution of this paradox for a single system lay in recognizing that the values assigned to degenerate quantities (for which there may be many states associated with a given eigenvalue) depended on the context provided by different incompatible nondegenerate quantities, of which the degenerate quantity could be regarded as a function in a well-defined mathematical sense.[19]

The route to an algebraic proof of nonlocality consisted in applying these ideas to the quantum mechanics of two separated systems. Even if a quantity, like a spin-projection along a particular direction, is nondegenerate (or maximal, as it is often referred to) when referred to the state space of a single particle, it becomes degenerate (nonmaximal) when considered with reference to the joint state space of the two particles. So could it be that giving value assignments to uniquely prescribed locally maximal quantities could lead to a Kochen-Specker paradox, which would force the conclusion that locally maximal quantities depend in their value assignments on a holistic context provided by incompatible quantities maximal in the joint state-space? If all this turned out to be the case we would have a demonstration of nonseparability in a well-defined mathematical sense. The question was posed by Jeffrey Bub in 1976,[20] but the answer that finally emerged in 1980 was that no such contradiction could be derived.[21] Nonseparability was not forced on pain of algebraic inconsistency.

In 1983 Peter Heywood and Michael Redhead took up the challenge and proved the following result:[22]

$$\text{VR} \wedge \text{FUNC*} \wedge \text{O-Loc} \wedge \text{E-Loc} \wedge \text{Determinism} \rightarrow \text{Contradiction.} \qquad (4)$$

The principles VR, FUNC*, O-Loc and E-Loc express the following ideas:

- VR (Value Rule): Values shall not be assigned to quantities that have a zero probability of turning up as measurement results.
- FUNC*: Functional relationships between maximal observables shall be preserved by value assignments.
- 0-Loc (Ontological locality): Locally maximal quantities are uniquely specified, independently of a holistic context.
- E-Loc (Environmental locality): Value assignments to locally maximal quantities for one particle are not affected by any physical manipulation of the environment of the remote particle.

Heywood and Redhead then showed that, modulo the other assumptions in the proof, 0-Loc and/or E-Loc must be violated. Essentially, Bub's idea had been to demonstrate a violation of 0-Loc on its own. This, as we have seen, could not be done, and the weaker conclusion was all that could be established. Probability enters the proof via VR, but there is no mention of statistics or correlation functions, so this first 'quasi-algebraic' nonlocality proof seemed a good deal more direct than the original Bell-type proofs.[23]

A completely new turn to the developing story of nonlocality proofs was introduced by Dan Greenberger, Michael Horne, and Anton Zeilinger in 1989[24] – henceforth referred to as GHZ – who produced a new algebraic proof of nonlocality, apparently quite unrelated to the Kochen-Specker paradox.[25] They considered a four-body decay problem in which an initial spin-one system decayed into two spin-one systems, each of which in turn decayed into two spin-1/2 systems. Denoting the four emerging particles by A, B, C, and D, GHZ considered the spin-projections on directions at right-angles to a common line of flight of the decaying particles. Let $A(\theta_A)$ be the spin-projection for particle A along direction q_A, similarly for $B(\theta_B)$, $C(\theta_C)$, and $D(\theta_D)$, then GHZ chose the quantum state so that the expectation value of the product of the four spin-projections had the following property:

$$\text{if } \theta_A + \theta_B + \theta_C + \theta_D = 0, \; <A(\theta_A) \bullet B(\theta_B) \bullet C(\theta_C) \bullet D(\theta_D)> = -1, \qquad (5)$$

$$\text{but if } \theta_A + \theta_B + \theta_C + \theta_D = \pi, \; <A(\theta_A) \bullet B(\theta_B) \bullet C(\theta_C) \bullet D(\theta_D)> = +1. \qquad (6)$$

GHZ then gave an argument to show that no local hidden-variable reconstruction of these expectation value results was possible, on pain of contradiction. The GHZ proof suffered from two defects: (1) it assumed determinism; (2) the proof, as sketched by GHZ was actually fallacious! (It effectively assumed that the measure of an uncountable infinity of intersections of measure-one sets was itself one!)[26]

Both these problems were set aside in the work of Rob Clifton, Michael Redhead and Jeremy Butterfield – hereafter CRB – who showed how to avoid the illegitimate assumption in the GHZ proof, and also generalized it to the framework of stochastic hidden-variable theories.[27] The proof proceeded in two stages. First CRB showed that, subject to appropriate locality assumptions, the satisfaction of (5)-(6) required the theory to be deterministic 'almost everywhere,' as the mathematicians put it.[28]

Secondly, CRB showed how to produce a contradiction using a finite (indeed rather small) number of settings for $\theta_A, \theta_B, \theta_C, \theta_D$.

One may note that the CRB contradiction involves a fixed setting for the fourth particle. So one suspects that a similar type of argument could be produced using a three-body rather than a four-body decay. That this is so has been shown in explicit detail by David Mermin.[29] Further discussion of experimentally realizable examples for the three-body and the four-body arrangements was given by Dan Greenberger, Michael Horne, Abner Shimony, and Anton Zeilinger.[30] These authors also independently derived the CRB contradiction.

It is also interesting to note that Mermin's proof can be generalized to the general case of N particles, and the question was posed, Does the proof go through even in the case of an infinite number of particles? This problem has been examined by Constantine Pagonis, Michael Redhead, and Rob Clifton[31] who fmd that the proof of nonlocality actually breaks down in the classical limit of an infinite number of particles. This confirms the intuition that the nonlocality under discussion is a peculiarly quantum-mechanical phenomenon.

5 The Relativistic EPR Argument

At first glance, the realist interpretations of quantum mechanics such as Bohm's offer many advantages over standard interpretations of the theory. In particular, they give a clear, intuitive picture of many potentially paradoxical. physical situations, such as the two-slit experiment and the phenomenon of barrier penetration. At the same time, their chief drawback – a form of nonlocality that seems, as we have claimed, to conflict with the constraints of relativity theory – is apparently shared by the standard 'antirealist' interpretations that reject hidden variables and assume completeness, as was demonstrated by the original Einstein-Podolsky-Rosen argument.

However, while the Bell argument that establishes nonlocality for realistic interpretations such as Bohm's has been formulated in a relativistic context,[32] there is no well-established relativistic formulation of the EPR argument. In the absence of such a formulation, it seems hasty to conclude that the tension between the standard interpretations and relativity theory is just as great as that between Bohmian interpretations and relativity. Clearly, if a relativistic formulation of EPR could be given that did not entail nonlocality, anti-realist interpretations would have an advantage over the Bohmian interpretation.

Let us now investigate the possibility of a relativistic formulation of the EPR argument. First, the standard nonrelativistic version of the EPR argument is reviewed and the problematics of translating it into a relativistic context are considered, paying particular attention to the need for a reformulation of the so-called reality criterion. Then, we introduce one such reformulated reality criterion, due to GianCarlo Ghirardi and Renata Grassi,[33] and show how it is applied to the nonrelativistic EPR argument. Next, we discuss the application of the new reality criterion in a relativistic context and point out a flaw in Ghirardi and Grassi's argument that appears to undermine their conclusion of peaceful coexistence between

quantum mechanics and special relativity. Finally, issues are engaged related to the evaluation of counterfactuals that reveal a hidden assumption of determinism in Ghirardi and Grassi's proof, while offering a way of salvaging their conclusion.

A relativistic version of the EPR argument must differ from the nonrelativistic version in two principal ways. First, the particle states must be described by a relativistic wavefunction. The details don't concern us here; we need only require that the wavefunction preserve the maximal, mirror-image correlations of the nonrelativistic singlet state. And indeed, the existence of maximal correlations in the vacuum state of relativistic algebraic quantum field theory has recently been demonstrated.[34] Second, the argument must not depend on the existence of absolute time ordering between the measurement events on the left and right wings of the system, for in the relativistic argument these may be spacelike separated. As it turns out, the nonrelativistic version of the argument does invoke absolute time ordering. To see how to get around this problem, we must briefly review the standard formulation of the incompleteness argument.

For EPR, a necessary condition for the completeness of a theory is that every element of physical reality must have a counterpart in the theory. To demonstrate that quantum mechanics is incomplete, EPR need simply point to an element of physical reality that does not have a counterpart in the theory. In this vein, they consider measurements on a pair of scattered particles with correlated position and momentum, but the formulation of the argument in Bohm,[35] in terms of a pair of oppositely moving, singlet-state, spin-1/2 decay products of a spin-0 particle, is conceptually simpler. In this case, the formalism of quantum mechanics demands a strict correlation between the spin components of the two spatially separated particles, such that a measurement of, say, the Z-component of spin of one particle allows one to predict with certainty the outcome of the same measurement on the distant particle. This ability to predict with certainty, or at least probability one, the outcome of a measurement is precisely the EPR criterion for the existence of an element of reality at the as-yet-unmeasured particle. By invoking one final assumption, a locality assumption stating that elements of reality pertaining to one system cannot be affected by measurements performed 'at a distance' on another system, EPR can establish that the element of reality at the unmeasured particle must have existed even before the measurement was performed at the distant particle. But the quantum formalism describes the particles at this point with the singlet state, and thus has no counterpart for the element of reality at the unmeasured particle. It follows that the quantum description was incomplete[36] Schematically,

$$\text{Quantum Formalism} \wedge \text{Locality} \rightarrow \sim \text{Completeness.} \tag{7}$$

Alternatively, if one assumes completeness, the argument may be rearranged as a proof of nonlocality:

$$\text{Quantum Formalism} \wedge \text{Completeness} \rightarrow \sim \text{Locality.} \tag{8}$$

The problematic assumption of absolute time ordering entered the argument in the reality criterion, which turns on the possibility of predicting with certainty the outcome of a measurement along one wing *subsequent* to having obtained the result of a measurement along the other. Of course, for spacelike separated events, notions like precedence and subsequence are reference-frame dependent, not absolute. So to translate the EPR argument to a relativistic context requires a modified criterion for the attribution of elements of reality that is not contingent on the time ordering of the measurement events.[37] Ghirardi and Grassi have undertaken to formulate just such a criterion, and thus to salvage the EPR argument in a relativistic framework. For the sake of clarity, we shall first describe how this criterion applies to the nonrelativistic version of the argument.

Ghirardi and Grassi's criterion rests on the truth of certain classes of counterfactual statements – statements of the form 'if ϕ were true, then ψ would be true' where the antecedent ϕ is in general known to be false. In particular, they wish to 'link... the attribution at a time *t* of the property corresponding to [observable *a* having value a] to the truth of the counterfactual assertion: if a measurement of [*a*] were performed at time *t*, then the outcome would be [a].'[38]

With this criterion in hand, Ghirardi and Grassi can now run the *nonrelativistic* EPR argument essentially as before. They assume a measurement of property *a* is performed on the right-hand particle at time t_R, yielding a specific result a. To ascertain whether an element of reality corresponding to property a = a′ exists at the left-hand particle, they must assess the truth of the counterfactual assertion: 'if I were to perform a measurement of property *a* at the left-hand particle at time t_L, I would obtain the result a′.' In the nonrelativistic case, the truth of this counterfactual assertion follows naturally from the presence of absolute time ordering. For if $t_R < t_L$, then the outcome of the right-hand measurement can be assumed to be the same in all of the 'accessible' (most similar) worlds used to evaluate the counterfactual, because it is strictly in the past of the counterfactual's antecedent. The strict correlation laws of quantum mechanics, also assumed to hold in all accessible worlds, then demand that the result of a measurement on the left wing also be fixed in all possible worlds (specifically, the laws require that a′ = -a). Thus the counterfactual is true, and an element of reality can be said to exist at the left-hand particle. From here, the argument unrolls in the usual way, and by supplementing this reality criterion with a locality assumption (they call it G-Loc, after Galileo), Ghirardi and Grassi can deduce that quantum mechanics is incomplete. Once again, we can represent their argument schematically by

Quantum Formalism ∧ G-Loc → ~ Completeness (9)

or

Quantum Formalism ∧ Completeness → ~ G-Loc. (10)

While these conclusions seem sound, the locality principle, G-Loc, bears further investigation. It reads: 'A system cannot be affected by actions on a system from which it is isolated. In particular, elements of physical reality of a system cannot be influenced by actions on systems from which it is isolated.' An examination of the structure of Ghirardi and Grassi's argument reveals that they make use not of the general principle stated but of a special case of this general principle, namely that elements of reality cannot be brought into existence 'at a distance.' It is this special case of G-Loc, call it ER-Loc (for elements of reality) that enters toward the end of the argument to establish that the measurement at the right wing could not have created an element of reality at the left wing and thus it must have existed prior to the measurement at the right wing, when the quantum formalism said the particles were in the singlet state. Thus they conclude that quantum mechanics is incomplete. All is well so far, but when one turns the argument around, assuming completeness and dispensing with locality, one must ask, can one be more precise as to which locality principle should be given up: the principle they label G-Loc, or the special case ER-Loc? Indeed it is the latter, for only it entered into the argument. As it turns out the distinction between G-Loc and ER-Loc does not affect their conclusions in the nonrelativistic case, because the conclusion they choose to highlight – the creation of elements of reality at a distance – is precisely one that does follow from dispensing only with ER-Loc.

In the relativistic case,[39] however, greater care must be taken with the statement of the locality principle, this time called L-Loc (after Lorentz by Ghirardi and Grassi), because a locality principle must enter at the very beginning of the argument as well as in the usual way at the end. The argument begins in the same way as in the nonrelativistic case, with the occurrence of a measurement on the right-hand side, but now the absence of absolute time ordering means the result of this measurement can no longer tacitly be assumed to be the same in all the accessible worlds used to evaluate the element-of-reality counterfactual at the left-hand side. Locality must be invoked to establish the independence of the outcome of the right-hand measurement from the occurrence of the measurement at the left. This done, Ghirardi and Grassi then demonstrate the existence of an element of reality at the left-hand side following the same reasoning as above. From here, the argument unrolls once again in the usual way and locality makes a second appearance in its familiar place at the end of the argument. In this way, Ghirardi and Grassi can again prove that standard quantum mechanics plus 'locality' implies incompleteness.

But there are two quite distinct cases of L-Loc that are actually being employed, one used in getting the argument started and the other appearing in the conclusion. Ghirardi and Grassi define L-Loc as the following: 'An event cannot be influenced by events in spacelike separated regions. In particular, the outcome obtained in a measurement cannot be influenced by measurements performed in spacelike separated regions; and analogously, possessed elements of physical reality referring to a system cannot be changed by actions taking place in spacelike separated regions.' As in the nonrelativistic case, it is not the general principle but rather the two special cases, call them OM-Loc (for outcome of measurement) and ER-Loc (again for element of reality), that are doing the logical work in their argument. OM-Loc affirms that the outcome of a measurement cannot be influenced by performing another measurement at a spacelike separation, while ER-Loc affirms that elements of reality cannot be created by performing a measurement at spacelike separation. Ghirardi and Grassi invoke OM-Loc at the beginning of the argument while applying the counterfactual reality criterion, as discussed above, and they invoke ER-Loc at the end of the argument, as they did in the nonrelativistic case. So if we write L-Loc = OM-Loc ^ ER-Loc, then, schematically, their argument looks like this:

Quantum Formalism ^ OM-Loc ^ ER-Loc → ~Completeness (11)

or

Quantum Formalism ^ Completeness → ~OM-Loc v ~ER-Loc. (12)

Ghirardi and Grassi now argue, in effect, as follows. Assuming OM-Loc we can again demonstrate from Completeness a violation of ER-Loc, i.e. Einstein's *spooky* action-at-a-distance *creating* elements of reality at a distance. But if we don't assume OM-Loc, then we cannot deduce a violation of ER-Loc. All this is quite correct, but the price we have to pay for *not* being able to demonstrate a violation of ER-Loc is precisely that we have to accept a violation of OM-Loc!

In other words, the relativistic formulation of the EPR argument does not help with the thesis of peaceful coexistence between quantum mechanics and special relativity, unless one argues that violating ER-Loc is more serious than violating OM-Loc from a relativistic point of view. This is hard to maintain since violating OM-Loc involves a case-by-case version of what Shimony refers to as violating parameter independence,[40] i.e. the independence of the probability of the outcome on one wing of the EPR experiment with respect to the parameters controlling the type of experiment being performed on the other wing. By analogy, violating ER-Loc is also a form of parameter dependence. Thus we find ourselves unable to agree with Ghirardi and Grassi's claim that in the relativistic context 'the conclusion that quantum mechanics implies...effects of parameter dependence, is not justified.'

To justify peaceful coexistence, we need to identify an additional assumption omitted from (12), which, if challenged, could undermine the inference. Recall that to run the argument in either the nonrelativistic or relativistic case, Ghirardi and Grassi must establish that the outcome of, say, the right-hand measurement is the same in all accessible worlds. With this established, the correlation laws of quantum mechanics imply that the outcome of the left-hand measurement is the same in all accessible worlds, and hence establish the truth of the counterfactual assertion about the left-hand measurement result that permits the attribution of an element of reality to the left-hand particle. In the nonrelativistic case, the constancy of the right-hand result is a natural consequence of the absolute time ordering as discussed above; in the relativistic case, it's not so simple. A premise akin to one that Michael Redhead, following Stapp, labels the Principle of Local Counterfactual Definiteness (PLCD) is needed to do this sort of work[41].

In the present case, PLCD may be taken to assert that the result of an experiment that *could* be performed on a microscopic system has a definite value that does not depend on the occurrence of a measurement at a distant apparatus. Ghirardi and Grassi implicitly assume that PLCD is licensed by their locality principle, for they invoke only OM-Loc to establish the constancy of the right-hand outcome in all accessible worlds. But we argue that PLCD does *not* follow directly from any typical locality principle, certainly not from one like OM-Loc, which asserts that the outcome obtained in a measurement cannot be influenced by measurements in spacelike separated regions. The reason is quite simple: while invoking locality may prevent measurements on the left-hand particle from influencing the result at the right and from breaking the constancy of the accessible worlds as far as the right-hand result is concerned, it does not prevent *indeterminism* from wreaking that sort of havoc. Intuitively, we can imagine that we run the world over again, this time performing the measurement on the left-hand particle. If we consider this left-hand measurement schematically as a point event with a backward light cone identical to that in the actual world, we are concerned with what will happen in the complement of the forward and backward light cones. Under indeterminism the events in this complement (the absolute elsewhere) simply cannot be assumed to remain the same.[42]

Thus it is maintained that Ghirardi and Grassi need both OM-Loc and an assumption of determinism to get their argument off the ground. Schematically, (12) is replaced by

Quantum Formalism ^ Completeness ^ Determinism → ~OM-Loc v ~ER-Loc. (13)

Thus , Determinism → PLCD → OM-Loc, and hence, by the disjunctive syllogism, we can

infer ~ER-Loc.

It seems then, that Ghirardi and Grassi's reformulation of the EPR argument in a relativistic context may be less general than they would have us believe, for its scope is limited to deterministic systems.

6 Conclusion

So where does the discussion of nonlocality in quantum mechanics rest in the light of the various proofs whose ideas we have been sketching?

Taking first the realist option, in which all observables have sharp values at all times[43], if we assume determinism or restrict the discussion to cases of strict correlation or anticorrelation where we can derive determinism on plausible assumptions, there seems very little scope for avoiding the conclusion of nonlocality for any 'realist' reconstruction of quantum mechanics. But experimentally we never observe absolutely *strict* correlations, so it may be argued that discussions of nonlocality should not deal with this ideal case, but should be based upon the experimentally realistic nonideal case. We are not convinced by this argument, since idealization is an essential aspect of any scientific theorizing. Be that as it may, the assumption of determinism for the nonideal case where correlations are less than perfect, may well be suspect. In such cases one is forced, as we have seen, to go to the stochastic hidden-variable framework, and much of the recent discussion has focused on the significance of violating the locality assumptions involved in this framework. Following Jon Jarrett[44] these break down into two classes.

1. Independence of the probability for a particular outcome of measurement on one particle conditionalized on the collective hidden variables, from the type of measurement being carried out on a remote particle. This is Shimony's parameter independence, which is already referred to above.

2. Independence of the probability for a particular outcome of measurement on one particle, conditionalized on the collective hidden variables, from the *outcome* of measurement carried out on a remote particle. This is what Shimony calls outcome independence.

Violation of (1) has been generally held to mark probabilistic causal dependence between the outcome of a local measurement and the 'setting' of a remote piece of apparatus, in prima facie conflict with standard interpretations of special relativity as prohibiting causal links between spacelike separated events. Violation of (2) has led to more controversy. It seems to demonstrate that the precise specification of the state of the source, or more generally of the particles before the measurements are undertaken, cannot be regarded as a common cause of the measurement outcomes. And this in turn suggests that such a common cause must be overlaid by a direct causal connection between the events consisting of the measurement results on distant particles obtaining in the way they actually do. We have challenged this view by proposing to interpret violation of (2), not in terms of a *combination* of common cause and direct cause, in the way proposed, but in terms of a noncausal direct dependence between measurement outcomes that lacks the necessary 'robustness' in respect of how the measurement results are brought about, to merit description as a causal connection.[45] This approach gives up the stochastic hidden-variable framework in favor of trying to understand the correlations between distant events in terms of a harmony-at-a-distance, or, as Shimony describes it,[46] a passion-at-a distance.

This idea ties in with, but is really quite distinct from, the fact that violation of (2) cannot be used to transmit signals between distant locations[47] since the necessity to recover quantum probability distributions by averaging over the hidden variables, means that we have no way of controlling the local marginal distributions for the results of measurement on one particle, by varying the type of measurement performed on distant particles. The no-signalling result is often cited to defuse the tension between nonlocality in quantum mechanics and the constraints of special relativity, but, at a deeper level, the 'nonrobustness' argument may be preferable.

Turning to the anti-realist option, we have examined Ghirardi and Grassi's attempt to reformulate the EPR argument in a relativistic context and argued that it is flawed by an ambiguously stated locality principle and a hidden assumption of determinism.

By making explicit the logical structure of their argument, the conclusion that in the relativistic case the existence of action-at-a-distance is not a valid deduction from the EPR argument has been undermined.[48]

This conclusion can, however, be rescued if an additional hidden assumption of determinism is exposed. Assuming indeterminism then, we claim to avoid the EPR inference to action-at-a-distance, and the concomitant challenge to peaceful coexistence between quantum mechanics and special relativity. Thus we end up agreeing with Ghirardi and Grassi, but for different reasons from the ones they present in their paper.

We should stress that we have been concerned in this discussion with violations of locality principles such as OM-Loc and ER-Loc, which actually figure explicitly or implicitly in the original EPR argument and its extension by Ghirardi and Grassi. Even if, as we have argued, violations of either ER-Loc or OM-Loc cannot be derived under an assumption of indeterminism, there remains of course the question of how to interpret the violation of outcome independence. Assuming completeness, outcome dependence famously follows. In the EPR set-up this means that when measurements are performed at spacelike separation on the two wings of the experiment, the results are mirror-image correlated. As one potentiality gets actualized on the left, say, how does this happen exactly in tandem with the opposite result on the right? Are we faced with a causal effect, namely result-to-result causation, so that peaceful coexistence with relativity is still challenged even if OM-Loc and ER-Loc are not violated? Similar answers involving no-signaling or nonrobustness may be provided as in the discussion above of the 'realist' option.

If these arguments are accepted, then our results concerning the EPR argument may be seen as closing *additional* gaps in the peaceful coexistence argument arising from the possibility of violating OM-Loc and ER-Loc over and above the fact of outcome dependence. But it must be stressed that the mysterious harmony of the result-to-result correlations remains arguably 'spooky' even if it does not involve causal dependence.

Another distinguishing feature of this harmony is its symmetrical character, quite unlike the asymmetry that one would normally want to ascribe to a causal connection. Shimony's phrase 'passion-at-a-distance' seems exactly the right one to capture what is going on, even if one concedes that the mystery of the EPR correlations is not eliminated merely by introducing an apt nomenclature.

For the antirealist the role of measurement is to actualize potentialities. If there are no measurements, then there are no actualities! This observation is particularly relevant to cosmological applications of quantum mechanics, where there is nothing 'outside' the universe to serve as a measuring device! So, from the cosmological perspective, one can argue that the realist option is after all to be preferred.

And here one can argue that the nonseparability approach, as exemplified in blocking the nonlocality proof by denying the 0-Loc principle, may in the end be the best way of understanding the peculiar features of entangled states in quantum mechanics.[49]

Many people find it difficult to see the distinction between violating 0-Loc (nonseparability) and violating E-Loc (action-at-distance). A simple example due to David Lewis[50] may help here. Consider someone with what she calls a bilocal hand. There is in reality just one hand that is manifested in two different places. If you shake hands with this curiously disabled person, her other hand will move in synchrony with the one you are shaking, not because there is an interaction between the two hands, but just because, in reality, there is only one hand, bilocally located! According to Lewis, this is an example of nonseparability as distinct from action-at-distance.

The aim of this chapter has been to produce *arguments* for invoking either indeterminism or holistic nonseparability in the interpretation of quantum mechanics. The theological implications of such interpretation should be clear. Indeterminism, as is claimed, allows 'room' for Divine Action on particular occasions, while allowing overall statistical laws to remain inviolate. Holism is an anti-reductionist thesis that shows how every element of the universe has for its ground of being the totality of the whole, which pantheists would want identify with God[51]. But too often the *scientific* arguments for these sort of claims are vague and woolly. We have tried to show that this need not necessarily be the case, although, as with much conceptual discussion, the arguments presented here are involved and intricate. The important point is that rigorous arguments are in fact possible on these important questions.

APPENDIX 1 – ELEMENTS OF QUANTUM MECHANICS[1]

The formalism of Quantum Mechanics (QM) is designed to accommodate two features of atomic and subatomic systems. First, the possible results of measuring certain physical magnitudes of such systems, are confined to a restricted set of possible values (real numbers). Secondly, it is in general not possible to predict, for any physical magnitudes what values will turn up on measurement, only the probability that any particular value from the set of possible values will turn up. Physical values that can be measured are called observables, and the specification of the probabilities of measurement results for observables depend on assigning to the system in question a state: in other words the state of a system is just an expression of the various probabilities, for all the observables, of the possible outcomes of measurement. The mathematical scheme for QM consists then in setting up a mathematical structure such that certain elements in that structure are associated with the states of the system and certain other elements are associated with the observables. Certain algorithms are then proposed which serve to answer our two basic questions:

1. What are the possible measurement results for any given observable?
2. For any given state and any given observable, what is the probability that one of the possible measurement results will actually turn up when a measurement is performed?

We shall refer to the algorithm that answers the first question as the Quantization Algorithm. The algorithm that answers the second question we shall refer to as the Statistical Algorithm.

The reader may wonder why we introduce two algorithms. The numbers generated by the quantization algorithm are usually those which turn up with non-vanishing probability according to the statistical algorithm. Or, to put it another way, if the probability of a certain measurement is always zero, that number cannot be the result of a measurement. But that is just wrong. Zero probability is quite different from impossibility. It is consistent, for example, with any finite number of occurrences in an infinite collective of outcomes, if we adopt the usual relative frequency interpretation of probability. The converse proposition that if certain measurement outcomes never occur, then the probability for the outcomes is zero, is of course correct, and the two algorithms must mesh is such a way that that result is satisfied.

In Dirac's formulation the mathematical structure is an abstract vector space V, equipped with an inner product which we write in the following way:

If α and β are any two vectors in V then the inner product of α and β is a complex number written as

$$< \alpha \mid \beta >$$

and we have $$< \beta \mid \alpha > = (< \alpha \mid \beta >)^*$$

where * denotes the complex conjugate.

We restrict ourselves to finite-dimensional *V*. States are associated with unit vectors in *V*, and observables are associated with so-called self-adjoint linear operators on *V*.

Consider a self-adjoint operator $\hat{Q}$. It possesses eigenvectors satisfying

$\hat{Q} \mid q_i > = q_i \mid q_i >$

(we follow Dirac by writing vectors by the symbol | >) where q_i is some real number known as an eigenvalue of $\hat{Q}$, and *i* runs from 1 to *N*, where *N* is the dimension of the vector space. The q_i may be all distinct in which case $\hat{Q}$ is said to be nondegenerate or maximal. If two or more of the q_i are equal in value, we speak of degeneracy. We shall interpret $\mid q_i >$ as the i^{th} eigenvector having an eigenvalue q_i. It is thus labelled by the index *i* not the numerical value of q_i. The set $\{ \mid q_i > \}$ can be chosen so as to provide a complete orthonormal set of vectors in *V*. This means that

$< q_i \mid q_j > = \delta_{ij}$ and for any vector $\mid \psi >$ in *V* we can write $\mid \psi >$ as some linear combination of the $\mid q_i >$.

Thus

$$\mid \psi > = \sum_{i=1}^{N} c_i \mid q_i >$$

where the complex coefficients are given by

$c_i = < q_i \mid \psi >$

We are now in a position to state the two algorithms.

The possible measurement results on $\hat{Q}$ are the eigenvalues of the associated operator *Q*.

The probability that *Q* will yield measurement results q_i is given by

$$\sum_{j \mid q_j = q_i} \mid c_j^2 \mid^2$$

where the notation Σ is used to denote summation over all values of *j*

$j \mid q_j = q_i$

for which $q_j = q_i$.
Thus for a maximal observable we just have $|c_i|^2$.

In this chapter we shall often use as examples of the finite-dimensional spaces required for describing the spin angular momentum of quantum-mechanical systems. Spin in QM is a vector quantity associated with the 'internal' degrees of freedom of a system. We denote the observables corresponding to the *X*, *Y* and *Z* components of a spin relative to a Cartesian reference frame by S_x, S_y and S_z respectively. The magnitude of the spin vector is denoted by $\boldsymbol{S}$.

So $\boldsymbol{S}^2 = S_x^2 + S_y^2 + S_z^2$.. The eigenvalues of $\boldsymbol{S}^2$ are s(s + 1) where s is an integer or half-integer. We choose units so that Plank's reduced constant is equal to unity. For a given value of s, $\boldsymbol{S}^2$ has a (2s + 1) - fold degeneracy which can be removed by specifying the eigenvalue of any one of the components of the vector $\boldsymbol{S}$. The eigenvalues for these components are simply *m*, where *m* has the value –s, -s + 1, .. s – 1, s, i.e. *m* has any one of 2s + 1 values spaced at regular intervals between –s and +s.

For s = ½, the possible values of m are ±½.

We consider now the problem of finding the eigenvalues and eigenvectors of the spin component along some direction which may be different from the z - axis.

For example take the positive x- axis. Define σ = 2 x $\boldsymbol{S}$, so the eigenvalues of σ_x, σ_y and σ_z are ±1. Then define $\mid \alpha > = \mid \sigma_z = +1 >$ and $\mid \beta > = \mid \sigma_z = -1 >$. Then denoting $\mid \sigma_x = +1 >$ by $\mid \gamma >$ and $\mid \sigma_x = -1 >$ by $\mid \delta >$, we obtain $\mid \gamma > = 1/\sqrt{2} (\mid \beta > + \mid \alpha >)$ and $\mid \delta > = 1/\sqrt{2} (\mid \beta > - \mid \alpha >)$. This is an example of superposition, the states $\mid \gamma >$ and $\mid \delta >$ are not simply the average components of $\mid \alpha >$ and $\mid \beta >$ but there now exists interference terms between $\mid \alpha >$ and $\mid \beta >$.

This is the case of the two-slit experiment. A beam of particles is filtered through a screen with two slits. The result is an interference effect between the result of allowing the first slit and the second slit to be opened.

We consider next the angular momentum of a composite system. For example, for a two particle system we employ a so-called tensor product to describe the states of the composite system. We shall now consider the problem of finding the eigenvalues and eigenvectors for the total system angular momentum $\boldsymbol{S}$ of the two particles in terms of the eigenvalues and eigenvectors of the spin angular momentum $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$ of the component particles separately. If s_1 and s_2 are the spin quantum numbers for the two particles, then the total spin quantum number s for the combined system can range in integral steps from $|s_1 - s_2|$ to $s_1 + s_2$. For a given *s*, the eigenvalues of the z-component of the total spin Sz can range in integral steps from –s to +s in the usual way. The problem of expressing $\mid \psi_z = m >$ in terms of $\mid \psi_{1z} = m' >$ and $\mid \psi_{2z} = m'' >$ where m' ranges from $-s_1$ to $+ s_1$ and *m''* from $-s_2$ to $+ s_2$ is solved in terms of the so-called Clebsch-Gordon coefficients.

For the singlet state of two spin - ½ particles we have s = 0 and the only value for *m* is 0. The resulting state vector is $\mid \psi$ singlet $>$ = $1/\sqrt{2} (\mid \alpha (1) > \mid \beta (2) > - \mid \beta (1) > \mid \alpha (2) >)$.

In this equation the arguments for the spin state vectors are used to distinguish the two particles. The singlet states of the total spin have the property of rotational invariance. These states all possess mirror symmetry in the sense that measuring the spin component on one particle enables us to predict that a subsequent measurement of the same spin component on the other particle will show the opposite value.

ENDNOTE

1. See M. L. G. Redhead, *Incompleteness, Nonlocality and Realism: A Prolegomenon to the Philosophy of Quantum Mechanics*, Clarendon Press Oxford, 2nd Edition, 1989, Chapter 1.

APPENDIX 2—GLOSSARY OF LOCALITY PRINCIPLES USED IN THIS CHAPTER[51]

Realist Interpretations

E-Loc: Local elements of reality cannot be affected by changes in a distant, spacelike separated environment.

O-Loc: Local elements of reality can be specified independently of a holistic context.

Antirealist Interpretations

G-Loc: Events cannot be affected by a distant measurement performed simultaneously.

L-Loc: Same as for G-Loc but with 'simultaneously' replaced by 'at spacelike separation.'

ER-Loc: Elements of reality cannot be created by distant measurements performed simultaneously/at spacelike separation – a special case of G-Loc/L-Loc.

OM-Loc: The outcome of a measurement at one location cannot be affected by a distant measurement performed at spacelike separation – another special case of L-Loc.

PLCD: The outcome of a measurement that could be performed has a definite value independent of whether or not another measurement takes place at a distant spacelike separated location. OM-Loc implies PLCD under an assumption of determinism, but not of indeterminism.

Parameter Independence: The *statistics* of measurement results at one location is independent of the parameters defining a measurement procedure at a distant spacelike separated location. OM-Loc is a case-by-case version of Parameter Independence. It is thus a logically stronger principle than the latter.

Outcome Independence: The *statistics* of measurement results at one location is independent of the outcome of measurements performed at a distant spacelike separated location.

ENDNOTES

1. This chapter is an updated and re-evaluated version of material originally presented in Michael Redhead, 'Nonlocality and Quantum *Mechanics*,' *Proceedings of the Aristotelian Society*, supp. vol.65 (1991): 119-40 and Michael Redhead and Patrick La Rivière, 'The Relativistic EPR Argument,' in *Potentiality, Entanglement, and Passion-at-a-Distance: Quantum Mechanical Studies for Abner Shimony*, vol. 2, R. Cohen, M. Horne, and J. Stachel, eds. (Dordrecht: Kluwer,1997). 207-15. cf Michael Redhead 'The Tangled Story of Nonlocality in Quantum Mechanics' in *Quantum Mechanics: Scientific Perspectives on Divine Action*, R. J. Russell, P. Clayton, K. Wegster-McNelly, J. Polkinghorne eds, (Vatican, Observatory Publications, 2001).

2. This view is defended, for example, in Michael Redhead 'Relativity, Causality and the Einstein-Podolsky-Rosen Paradox: Nonlocality and Peaceful Coexistence,' in *Space, Time and Causality*, R Swinburne, ed. (Dordrecht: Reidel, 1983):151-89.

3. J. R. Lucas, *The Freedom of the Will* (Oxford: Oxford University Press, 1970).

4. However, cosmological considerations tell against such an approach and may favour the realist-holistic treatment, as I advocate in *From Physics to Metaphysics* (Cambridge: Cambridge University Press, 1995).

5. Indeed, the 'range' of a wavefunction may be much longer than the 'range' of interaction, as in the case of so-called Efimov states in nuclear physics.

6. Albert Einstein, Boris Podolsky, and Nathan Rosen, 'Can Quantum-Mechanical Descriptions of Reality be Considered Complete?', *Physical Review 47* (1935): 777-80.

7. In the case of indistinguishable particles question (A) becomes more controversial since the observables of quantum mechanics are restricted to symmetric functions of the particle labels. See Steven French and Michael Redhead, 'Quantum Physics and the Identity of Indiscernibles,' *The British Journal for the Philosophy of Science* 39 (1988): 233-46.

8. John Bell, 'On the Einstein-Podolsky-Rosen Paradox,'*Physics* 1 (1964): 195-200.

9. See David Bohm, *Quantum Theory* (Englewood Cliffs, N J: Prentice-Hall, 1951), 614-9.

10. Bell, 'On the Einstein-Podolsky-Rosen Paradox.'

11. Henry Stapp, 'S-Matrix Interpretation of Quantum Theory,' *Physical Review D* 3 (1971): 1303-20, and Phillipe Eberhard, 'Bell's Theorem without Hidden Variables,' Il *Nuovo Cimento* 38B (1997): 75-80.

12.This point was particularly stressed by Redhead, 'Relativity, Causality and the Einstein Podolsky-Rosen Paradox'; idem, *Incompleteness, Nonlocality, and Realism: A Prolegomenon to the Philosophy of Quantum Mechanics*, 2nd revised impression (Oxford: Clarendon Press, 1989). But meanwhile Arthur Fine, 'Hidden Variables, Joint Probability and the Bell Inequalities', *Physical Review Letters* 48 (1982): 291-5, had claimed to show that satisfaction of the Bell inequality was a *sufficient* condition for JD to obtain. Fine's result has been generalized to consider the conditions under which any set of experimental probability distributions can be returned as marginals on a common probability space. For a complete discussion of what has been achieved in this direction, see Itamar Pitowsky, *Quantum Probability – Quantum Logic* (Berlin: Springer, 1989). Fine had claimed, contra Redhead, that the Stapp-Eberhard 'proof' must have a concealed premise equivalent to JD, so the violation of the Bell inequality could be imputed to a failure of JD, rather than any unacceptable violation of locality. The situation here was clarified in the work of George Svetlichny, Michael Redhead, Harvey Brown, and Jeremy Butterfield, 'Do the Bell Inequalities Require the Existence of Joint Probability Distributions?', *Philosophy of Science 55* (1988): 387-401, who pointed out that Fine's mathematics showed only that satisfaction of the Bell inequality was sufficient for the existence of a classical probability space returning the measured experimental joint distributions as marginals. This space would have relative frequency models satisfying JD, but there was no argument to show that the relative frequencies generated.in the real world should conform to these purely mathematical models. So it was perfectly consistent to claim both the denial of JD and the satisfaction of the Bell inequality, and Fine's maneuver failed to vitiate the Stapp-Eberhard proof. Another telling objection against giving up JD is that,for states involving strict correlation or anticorrelation, joint distributions for incompatible observables on the *same* particle is a direct consequence of the correlation (anticorrelation), so a general prohibition on JD as applied to these important special cases, is not a viable option. Furthermore, Fine seems committed to the view that altering the experimental set-up for one particle from a Bell-conforming to a Bell-violating setting can mysteriously cause JD to obtain or not obtain on the distant particle. For full discussion see Svetlichny et al., 'Do the Bell Inequalities Require the Existence of Joint Probability Distributions?'.

13. Stapp attempted to modify his proof to meet my objection, but it has been claimed by Rob Clifton, Jeremy Butterfield and Michael Redhead, 'Nonlocal Influences and Possible Worlds – A Stapp in the Wrong Direction,' *The British Journal for the Philosophy of Science* 41 (1990): 5-58, that none of Stapp's later proofs can be made to work, and that the success of the Stapp-Eberhard approach in avoiding JD was offset by its involvement with determinism. See Redhead, *Incompleteness, Nonlocality and Realism*, 90ff., for a detailed discussion. This whole topic of evaluating counterfactuals under an assumption of indeterminism has been the subject of a long-running debate between Stapp and myself. The most up-to-date list of references on the topic can be found in Rob Clifton and Michael Dickson, 'Stapp's Algebraic Argument for Nonlocality,' Physical Review A 49 (1994): 4251-6; and Henry Stapp, 'Reply to `Stapp's Algebraic Argument for Nonlocality', *Physical Review A* 49 (1994): 4257-60.

14. John Clauser and Michael Horne, 'Experimental Consequences of Objective Local Theories,' *Physical Review D*10 (1974): 526-35.

15. John Bell, 'Introduction to the Hidden Variable Question,' in *Foundation of Quantum Mechanics: Proceedings of the International School of Physics 'Enrico Fermi '*, Course 49, Bernard d'Espagnat, ed. (New York: Academic Press, 1971), 171-81.

16. I here correct the statement made in Michael Redhead, 'Undressing Baby Bell,' in *Harre and his Critics*, R. Bhaskar, ed. (Oxford: Blackwell, 1990): 122-8. See also Jeremy Butterfield, 'Bell 's Theorem: What It Takes,' *The British Journal for the Philosophy of Science* 43 (1992): 41:83.

17. The discussion of the locality assumptions required in the derivation of Bell inequalities in the stochastic hidden-variable framework was given a very careful formulation by Jon Jarrett, 'On the Physical Significance of the Locality Conditions in the Bell Arguments,'Nous 18 (1984): 569-89. Two locality conditions were introduced, which are now usually referred to as parameter independence and outcome independence (following Abner Shimony), These will be defined and discussed more fully below.

A further twist to the argument was given in 1976 by Patrick Suppes and M. Zanotti, 'On the Determination of Hidden-Variable Theories with Strict Correlation and Conditional Stochastic Independence of Observables,' in *Logic and Probability in Quantum Mechanics*. P. Suppes, ed. (Dordrecht: Reidel, 1976), 445-55, who showed that conditional stochastic independence was anyway incompatible with the strict mirror-image correlations built into the singlet spin-state used in the original 1964 proof , except in the degenerate case when determinism was restored. In point of fact it can be argued that we do not need to start with the stochastic hidden-variable framework, in order to show that for the case of strict correlation or anticorrelation, locality implies determinism. We can use the EPR argument to justify the fact that the measurement results for the distant particle cannot evolve in time stochastically, since the predicted results for the distant measurement hold good for all times subsequent to the first measurement, so stochasticity fails for the distant spins after the first measurement, but, by locality, this lack of stochasticity cannot be induced by the first measurement, so there is never any stochasticity, i.e., determinism must be true. The argument was first given in Redhead, 'Undressing Baby Bell' (which was written and widely circulated in 1981). See also Geoffrey Hellman, 'EPR, Bell and Collapse: A Route Around 'Stochastic' Hidden-Variables,'*Philosophy of Science* 54 (1987): 558-76.

18. Simon Kochen and Ernst Specker, 'The Problem of Hidden Variables in Quantum *Mechanics,' Journal of Mathematics and Mechanics* 17 (1967): 59-87.

19. Cf. Bas van Fraassen, 'Semantic Analysis of Quantum Logic,' in *Contemporary Research in the Foundations and Philosophy of Quantum Theory*, C. A. Hooker, ed. (Dordrecht: Reidel, 1973), 80-113.

20. Jeffrey Bub, 'Hidden Variables and Locality,' *Foundations of Physics* 6 (1976): 511-25.

21. See William Demopoulos, 'Locality and the Algebraic Structure of Quantum Mechanics,' 119-44, Jeffrey Bub, 'Comment on 'Locality and the Algebraic Structure of Quantum Mechanics' by William Demopoulos,' 149-53, Paul Humphreys, 'A Note on Demopoulos's Paper 'Locality and the Algebraic Structure of Quantum Mechanics',' 145-7, all in *Studies in the Foundations of Quantum Mechanics*, Patrick Suppes, ed. (East Lansing, Mich.: P S A, 1980).

22. See Peter Heywood and Michael Redhead, 'Nonlocality and the Kochen-Specker Paradox,' *Foundations of Physics* 13 (1983): 481-99. A similar approach was arrived at Independently by Simon Kochen, although never published (private communication).

23. The Heywood-Redhead proof has since been improved in a number of directions. Firstly, it should be noted that we found it necessary to introduce value assignments for locally nonmaximal quantities to make the proof work. Allen Stairs, 'Quantum Logic, Realism, and Value Definiteness,' *Philosophy of Science* 50 (1983): 578-602, showed that a more elegant proof could be given under weaker conditions, requiring only consideration of value assignments to locally maximal quantities. The Stairs proof was further examined and more carefully formulated in respect of all relevant presuppositions

by Harvey Brown and George Svetlichny, 'Nonlocality and Gleason's Lemma. Part 1. Deterministic Theories,' *Foundations of Physics* 20 (1990): 1379-88. The work of Brown and Svetlichny brought the quasi-algebraic proofs of nonlocality to the same degree of sophistication as the Bell proofs, but, as we have stressed, with the added advantage of not requiring analysis of potentially complex correlation experiments, as in approach.the case of the Bell approach.

24. Dan Greenberger, Michael Horne, and Anton Zeilinger, 'Going Beyond Bell's *Theorem,'* in *Bell's Theorem, Quantum Theorem and Conceptions of the Universe*, M. Kafatos, ed. (Dordrecht: Kluwer, 1989), 69-72.

25. But see David Mermin, 'Simple Unified Form for the Major No-Hidden-Variables Theorems,' *Physical Review Letters* 65 (1990): 3373-6.

26. I stress that the GHZ proof, as published, is only a descriptive sketch, but in my opinion no proof of the sort described is rigorously possible.

27. Rob Clifton, Michael Redhead, and Jeremy Butterfield, 'Generalisation of the Greenberger-Horne-Zeilinger Algebraic Proof of Nonlocality,' *Foundations of Physics* 21 (1991): 149-84.

28. Compare the Suppes-Zanotti result referred to above in endnote 17 for the two spin-1/2 correlations.

29. David Mermin, 'Quantum Mysteries Revisited,' *American Journal of Physics* 58 (1990): 731-4; idem, 'Simple Unified Form for the Major No-Hidden-Variables Theorems.' Mermin's proof has been reworked with more careful attention to all concealed assumptions by Adam Stocks and Michael Redhead, 'A Value Rule for Non-Maximal Observables,' *Foundations of Physics Letters* 9 (1996): 109-19.

30. Dan Greenberger, Michael Horne, Abner Shimony, and Anton Zeilinger, 'Bell's Theorem without Inequalities, *American Journal of Physics* 58 (1990): 1131-43.

31. Constantine Pagonis, Michael Redhead, and Rob Clifton, 'The Breakdown of Quantum Non-Locality in the Classical Limit,' *Physics Letters* A 155 (1991): 441-4.

32. S. J. Summers and R. Werner, 'The Vacuum Violates Bell's Inequalities,' *Physics Letters A* 110 (1985): 257-9; L.J. Landau, 'On the Non-Classical Structure of the Vacuum,' Physics Letters A 123 (1987): 115-8.

33. GianCarlo Ghirardi and Renata Grassi, 'Outcome Predictions and Property Attribution: the EPR Argument Reconsidered,' *Studies in History and Philosophy of Science* 25 (1994): 397-423.

34. Michael Redhead, 'More Ado About Nothing,' *Foundations of Physics* 25 (1995): 123-37. The argument actually applies more generally to any state of bounded energy; see also, 'The Vacuum in Relativistic Quantum Field Theory,' in PSA 1994, vol.2, D. Hull, M. Forbes and K. Okruhlik, eds. (East Lansing, Philosophy of Science Association, 1995), 77-87. For a unified treatment of the relativistic EPR and Bell arguments, see Michael Redhead and Fabian Wagner, 'Unified Treatment of EPR and Bell Arguments in Algebraic Quantum Field Theory *'Foundations of Physics Letters* 11 (1998): 111-25.

35. Bohm, *Quantum* Theory (op.cit.).

36. I follow here a streamlined version of the EPR argument, as introduced in Redhead, 'Relativity, Causality and the Einstein-Podolsky-Rosen Paradox,' and Geoffrey Hellman, 'EPR, Bell and Collapse: A Route Around 'Stochastic' Hidden-Variables,' *Philosophy of Science* 54 (1987): 558-76. Historically this version of the argument seems to have been known to Einstein. See, for example, Arthur Fine, *The Shaky Game* (Chicago: University of Chicago Press, 1986). This is also the version used in the paper under discussion, Ghirardi and Grassi, 'Outcome Predictions and Property Attribution.'

37. An analysis of the EPR argument using counterfactuals (as distinct from purely conditional reasoning), though not specifically in the context of a relativistic reformulation, has been undertaken by Linda Wessels, 'The EPR Argument: A Post-Mortem', *Philosophical Studies* 40 (1981): 3-30. She seeks to uncover the full logical structure of the EPR argument by formulating the original paper's somewhat ambiguous reality criterion in precise modal terms. Among four possible modal readings of the EPR reality criterion, she lists a counterfactual reading similar to Ghirardi and Grassi's, which is the one I also adopt in the present work.

38. In order to evaluate the truth of such statements, they call on David Lewis, *Counterfactuals* (Cambridge: Harvard University Press, 1973): Let us denote the counterfactual 'if φ were true, then ψ would be true' as '$\varphi \, \Sigma \, \psi$' Then Lewis proposes the following truth condition: $\varphi \, \Sigma \, \psi$ true at world w if either (i) there are no possible worlds at which j is true or (ii) some world where both φ and ψ are true is more similar ('closer') to w than any world in which φ is true and ψ is false. Obviously one has to specify the possible worlds one is taking into account; this is done by assigning to each world w a set of worlds Sw called the sphere of accessibility around w.

39. A discussion of the EPR set-up in a relativistic context has also been provided by G.J. Smith and R. Weingard, 'A Relativistic Formulation of the Einstein-Podolsky-Rosen Paradox,' *Foundations of Physics* 17 (1987): 149-72. They argue that any relativistic formulation of EPR should employ a relativistic correlated state. They derive such a state and demonstrate the relativistic invariance of the correlations. However they fail to pursue the analysis beyond the existence of the correlations, i.e. to develop the full EPR argument.

40. See Abner Shimony, *Search for a Naturalistic World View*, vol. II (Cambridge: Cambridge University Press, 1993), 138, for his preferred terminology in this matter. For more discussion of this terminology see below.

41. Redhead, *Incompleteness, Nonlocality*, and *Realism*, 92.

42. This claim is not uncontroversial, however, for Lewis himself has argued that the events in the complement can be assumed to be fixed; thus for Lewis, OM-Loc *does* licence PLCD; David Lewis, private communication. See also David Lewis, Philosophical Papers, vol. II (Oxford: Oxford University Press, 1986). His argument turns on a dual reading of the 'might' counterfactual implicit in our description of 're-running' the world: if I were to run the world over again and perform the left-hand measurement, the right-hand outcome might be different than it was in the

actual world. This 'might,' he argues, could be read either as 'would be possible' or as 'not would not,' but that the first reading does not contradict the negation of the second reading. I am not enamoured of this slippery semantic solution to the problem that is forced on Lewis by his insistence on including events in the absolute elsewhere in assessing the similarity relation between worlds.

43. We exclude the case where determinism applies also to our free will in choosing the outcomes to be measured. We rely here on John Lucas's *The Freedom of the Will* (op. cit.). See also John Bell's paper 'Free variables and Local Causality in Speakable and Unspeakable in Quantum Mechanics (Cambridge: Cambridge University Press, 1987), J.S. Bell:100-04.

44. Jon Jarrett, 'On the Physical Significance of the Locality Conditions in the Bell Arguments', Nous 18 (1984): 569-89.

45. Michael Redhead, 'Relativity and Quantum Mechanics :Conflict or Peaceful Coexistence', *Annals of the New York Academy of Sciences* 480(19860:14-20; idem, *Incompleteness, Nonlocality, and Realism:* idem., 'Nonfactorizability, Stochastic Causality, and Passion-at a-Distance,' In *Philosophical Consequencies of Quantum Mechanics*, J. T. Cushing and E. McMullin, eds. (Notre Dame University Press, 1989):145-53; idem., 'Propensities, Correlations, and Metaphysics' *Foundations of Physics* 22 (1992): 381-94. For a careful discussion of these issues, see also Jeremy Butterfield, 'David Lewis Meets John Bell', *Philosophy of Science* 59 (1992): 26-43.

46. Abner Shimony, ' Controllable and Uncontrollable Nonlocality' in *Proceedings of the International Symposium :Foundations of Quantum Mechanics in the Light of New Technology* S. Kamafuchi et al. eds. (Tokyo: Physical Society of Japan,1984), 225-30.

47. For further discussion of the no-signalling results see Redhead, *Incompleteness, Nonlocality, and Realism*, 113ff, and other references detailed there, and also Phillipe Eberhard and R. Ross, 'Quantum Field Theory Cannot Provide Faster-Than-Light Communication,' *Foundations of Physics Letters* 2 (1988): 127-49.

48. Note in particular that my version of the relativistic EPR argument makes no reference to the contentious issue of relativistic state-vector collapse. In this sense my discussion transcends any proposed resolution of that notorious problem.

49. This is the approach defended in, for example, in Michael Redhead, *From Physics to Metaphysics* (op.cit.).

50. Private communication.

51. Of course, indeterminism can also be combined with holism.

52. The formulation of the Locality Principles in this appendix is adapted to the precise role they play in the arguments related to realist interpretations on the one hand and antirealist interpretations on the other, as presented in the present account.

CHAPTER 6 – RELIGION AND REVEALED TRUTH

There is a view, defended in extreme form, in the Abrahamic Religions. It counts as revealed truth its own interpretation of religion as afforded by the accounts in its sacred scriptures, and thus runs counter to those forms of revealed truth espoused by the others. These other truths are to be rejected and attacked and extirpated as heretical. These views are defended by extreme fundamentalists on all sides and are used to define their own version of theism and justify any and all utterances and actions.

These extreme views account, in part, for the bad press of religion, the idea that because one view is right and everything else is wrong, everything else must be converted (*however this is achieved*) to the one true view. Examples of these views are those of extreme evangelical Christians on the one hand and extreme Islamic fundamentalists on the other, and in fact we are seeing the emergence of extreme Hindu behaviour in India, which is erupting into violence. Indeed the Abrahamic religions religions are all beneficiaries of conversions with the sword. It may be argued that this view is (ironically) not dissimilar to the views put forward by many atheists who point to some inconsistency (internally or with external evidence) in a faith and then say that none of it has any foundation in logic and therefore *must* be abandoned. We have already discussed the idea that truth may outrun provability, but this is not to say that *everything* which is not amenable to proof is true, but it can not be dismissed for this reason alone.

While we have already considered the idea that truth can outrun provability in the scientific or mathematical sense; we might also consider this proposition in a wider sense. Religious revelation comes down to us from very early (and to the modern western view, backward) times. While some religions have adapted to changes in time and location, others have remained firmly rooted in the time and location of their origin – this notwithstanding that their adherents have migrated into a completely different host culture and way of life.

Many of the earliest records and foundations are contained in the body of religious writings themselves (whose own origins are not clear, that is to say who actually wrote them and exactly when) and this origin is obscured by the lack of support by external evidence or corroboration. Although modern scholarship and discovery may cast some further light, this enquiry is often hindered by accusations of blasphemy. When we consider the evaluation of historical events, we might consider the lawyer's ideas of proof, in that we have the evidence of witnesses from ancient times asking to be heard.[1] In the same way that mathematical proof is based upon a set of rules, so a lawyer's proof is subject to a technical set of rules. Evidence and proof in the courts may be illuminated by the following story sometimes quoted in the introduction to evidence textbooks:

Judge (to counsel): Am I not to hear the truth?

Counsel (in reply): No, my lord, you are to hear the evidence.

Evidence of eyewitnesses is often suspect because they see what they expect to see, and the scope of vision, like the lens of a camera, is limited to what is actually seen – not what comes before or after or is out of the field of vision. Written evidence compiled after the event is treated with considerable caution – the longer after, the more caution. Second-hand evidence (or hearsay) is not admissible, and this rule is further extended by the rule against implied hearsay. Then we should ask if we should only have reliable evidence presented to us. In the case of scientific evidence it is suggested that this is so[2] – but how will reliability be established? Today's reliable evidence may be discredited tomorrow. Even evidence by a first hand observer or participant in events would require to be tested by cross-examination. Perhaps the best modern examples of the fallibility of written accounts of events might be the memoirs of politicians. But this does not mean that every written account should be disregarded, rather that it should be read with realisation of its fallibility and the reasons why it was written. How are we to find the truth? Pilate asked, 'What is truth?'[3]. But since Latin has no articles, perhaps his question should better have been interpreted as,'What is the truth?'

These ideas do not consider the standard of proof. When we considered Gödel's Theorem earlier, we considered the idea of mathematical proof or, in other words, absolute proof. But the Common Law lawyer's standard of proof is either, beyond reasonable doubt, or on the balance of probabilities. Both of these standards admit less than cast iron certainty, in the first case by the definition of what is reasonable, and in the second case by the definition of balance. Both of these standards, thus, contain subjective elements[4], so that indeed something may be true, and unable to be proved, and also may be false but yet satisfying the demands of legal proof. But we need to think about the different functions of evidence and proof, lawyers are exclusionary, others more inclusive. Lawyers decide, historians and scientists conclude[5], therefore legal standards of proof may not always be appropriate.

However, we must not forget that ideas and writings are the product of their own time and location, but that they are often interpreted and evaluated misleadingly out of their original context. There are many examples where the adoption of a particular religion or sect has mapped on to fault lines in the population, and operated to reinforce such divisions; these examples range from the earliest times to modern times – particularly where such religion or sect has been spread by conquest or become identified with the state. One may question the motives of those who encouraged this, whether they were in pursuance of purely political or secular goals. Interestingly those countries, which are or appear to be heavily dominated by their religion (with the exception of Israel), do not have a high rate of immigration, but rather the reverse.

Of course we recognise that defending our faith is what is at stake here, but we do not think that that demands a criticism of other faiths. While, perhaps, it is too much to ask that all faiths be respected, it is not too much to ask that they all be accorded courtesy. In many cases beliefs are now so deep seated that they are incapable of eradication by argument. We regard a demonstration of how we behave to others as the best test of our own faith. We think that our faith should be strong enough to allow for any discussion and disagreement.

One test of this strength might be how one faith can co-operate with another. Modern western examples are the invitation to a Unitarian Christian minister to preach in a mosque in Oxford[6], and the holding of multi-faith services in the college chapel of Harris Manchester College Oxford[7]. A very early eastern example is that afforded by the Nestorians who helped Indian Buddhists translate their sutras into Chinese, and their possible influence on the development of Buddhism in Tibet[8]. Arab Christian theologians at the turn of the first millennium proposed discussion with Muslims, not on the basis of quoting from the sacred writings which they did not acknowledge, but on the basis of argument, reason and deduction. At the same time Muslim scholars did their best to understand the differences between the various Christian churches, highlighting the similar basic beliefs, while acknowledging their different theologies[9]. At the very heartland of the Abrahamic religions in Jerusalem, Muslims and Christians pray at the Greek Orthodox shrine of St George at Beit Jala, and modern Syria provides a good example of how Muslims and Christians can live together in amity[10]. It is to be hoped that as the major faiths continue to evolve, further opportunities for co-operation are found and acted upon.

Another reason for criticism of religion is the behaviour of the faithful; a common example here being the crusades. In this connection we must remember that this period of history was unbelievably violent, the cultural differences between Europe and the Eastern Mediterranean were wide and atrocities were committed on both sides. If we try to view these times through the lenses of the present day, our interpretation becomes distorted. As we turn the pages of the history books we can see examples down to the present day, (some rather smaller than the crusades) of atrocities committed in pursuit of an ideal. All faiths may be represented in these examples. This tit-for-tat argument solves nothing. Does this mean that all these faiths are flawed, or does it mean that those individuals are flawed, or can the genuineness of motive excuse all thoughts and actions?

It may be said that nearly everyone knows an individual who is a good person and who subscribes sincerely to a faith and behaves accordingly, and whose actions might be well perceived from any viewpoint. In some of these cases the person's faith may help them over otherwise insurmountable problems or improve the quality of their lives in some way. In these cases their faith is right for *them*[11]. However the question of freedom of religion (or more properly freedom of religious action) will continue to be a thorny one, and although freedom of religion is enshrined in many modern western societies, it may be wondered what would be the outcome if this principle were tested seriously. In the case of any conflict between the requirements of religious principles and the law of the land, which should prevail? To some extent this is already being tested in the areas of abortion and adoption and sexual orientation; compromises are sought, although the very firm holders of very firm beliefs remain unyielding. Some cases concerning the wearing of religious symbols appear to give rise to some anomalies, with the application of seemingly blanket rules.

This leads on to the question of special treatment on account of faith. Examples of this include special faith education and exceptional treatment under the law. Faith education is sometimes seen as divisive and having the effect of reinforcing and exacerbating those existing fault lines in societies which already have problems which can be traced to religion. On the other hand many faith schools have the reputation for providing a disciplined environment to the benefit of their students. This benefit has to be weighed against the criticism that the dogmas of the particular faith are being taught to pupils who are too young to choose a faith for themselves, in other words that the pupils are being indoctrinated. The tension between secular and faith based education is difficult to resolve especially in the matter of state provision, and the arguments on both sides are persuasive. The provision of faith-based education in the independent sector does not really provide a solution to this problem, especially when the arguments are obscured by accusations of elitism.

Accountability to external laws over which we have no control is considered unacceptable in western society in the current age. Many medieval kings in Europe found themselves in conflict with the Church over just this question and the resolution and supremacy of the secular law of the land took many generations to achieve. Folk memory of the threat of the Spanish Inquisition and England's gallant defence against the Spanish Armada is still alive[12], and contributes to the fear of religion. We can see a resurgence of this fear of external religion when we look at the behaviour of some Muslim extremists. It must be remembered that many Muslims hark back to the flowering of an Islamic civilization in the Middle Ages and, regretting its passing, would like to see its reinstatement[13]. In the West we see ourselves as members of our particular Nation first and members of our religion second; many Muslims see themselves as Muslims first and members of their Nation second. Many in the West find this worrying, and many Muslims see themselves as victimised – so we can see a religious divide forming with entrenched positions on either side.

Those of us who come from a secular Common Law tradition are not comfortable with anything, which allows inequality (or preferential treatment) before the law for whatever reason. However we must remember that the great milestones in this area were of their time. The barons who forced King John to sign Magna Carta were only concerned that those provisions of equality before the law applied to themselves and their class vis-à-vis the king. Similarly the Founding Fathers of the United States who subscribed to the declaration that all men were born equal were slave owners. Trivial modern examples of this differential treatment before the law are the exceptions to laws requiring motorcyclists to wear crash helmets (enabling the wearing of turbans where required by the wearer's religion). More serious is the expectation that a country is expected to alter its own law to take account of minority religious sensibilities – for example the fatwa against Salman Rushdie over *The Satanic Verses*, or the riots in protest against the publication by a Danish journal of cartoons depicting the Prophet Mohammed.

It has been suggested that elements of Sharia Law be incorporated into English Law (a suggestion which has provoked some outcry although to some extent this has already happened by the provision for legally binding arbitration in civil cases). Western society feels uncomfortable with some Middle Eastern and Asian treatment of women and women's issues. However in societies which have been historically diverse, the application of their own civil (as opposed to criminal) legal rules to each section of society has been considered normal, and there is no doubt that modern western legal systems may have to adjust themselves to accommodate the new diversity of populations with which they find themselves. There are already good examples of absorbing religious law into the law of the land from Roman times with the *ius civile* and the *ius gentium*, to the overriding powers of injunction by civil courts in the USA, judicial orders in France, the incorporation of Islamic financial instruments, and so on. In the United States there is also the incorporation of tribal sovereignty for Native Americans[14].

The spectre of a state within a state (perhaps an exaggeration, when this term is applied to a section of a population which is not amenable to the general law of the land) has haunted many rulers down to the present time. Either the minority, increasingly holding itself apart, will cease to be identified with the remainder of the nation, or will be perceived as being protected by a stronger external power and provoke jealousy of its wealth or privileges. Examples range from Jewry from ancient times down to the last century in Europe, Christians in the Roman Empire before the Emperor Constantine, the Catholic Church in Europe from the Middle Ages to the Reformation (and, perhaps beyond), the Knights Templar and the Teutonic Knights in the Middle Ages, Middle Eastern and Asian Christians after the rise of Islam, the Palestine Liberation Organization in Jordan.

Western Society has, until recently, been homogeneous. Those who have felt unhappy in their surroundings have been encouraged to seek new habitation, or flee persecution. Some examples of diverse societies are the Caliphates and the Ottoman Empire, both of which had large Jewish and Christian minorities, which did indeed diminish with the passing of time. This results from the time of the original Arab Muslim conquests, when the conquerors were in the minority, and made considerable use of the original well-educated Jewish and Christian inhabitants. This is not to say that these minorities had equal treatment under the law, but were subjected under *dhimmitude* to some civil disabilities, extra taxes and dress codes. Within these restrictions they were largely left alone or, at least, were not persecuted with the vigour with which minorities were persecuted in Christendom. But we must remember that all these people were living in a largely common Middle Eastern culture of empire rather than in the modern idea of the nation-state – even though there may be some lessons, which we may note.

Western Europe, on the other hand, developed a homogeneous culture based upon a feudal structure and the Roman Catholic Church. Deviance from the tenets of the church was ruthlessly suppressed. When Protestantism gained the upper hand, this trend continued. In more recent times Jewish immigrants integrated successfully into western culture, while preserving their religion. Up until the middle of the last century the majority of migrants to the West were from European or European style cultures. The second half of the last century saw large scale migration to the West from the rest of the world. These immigrants brought their own culture and religion with them. Some groups of immigrants have achieved higher profile than others, some have adapted and integrated to the host cultures, others have tried to preserve their old culture and way of life as much as they can.

Diversity is now with us in the West, as it has never been before, and the close relation between culture and religion gives us, in the West, issues to confront for the first time. Many of the new immigrants are Muslim and have been brought up to consider the world as *Dar al Islam* (sphere of Islam) and *Dar al Harb* (sphere of war), this largely independent of race or nationality. Attempts to treat the populations of the various western countries as homogeneous may be going against the grain of reality, and perhaps we shall have to look to see if there is anything we can learn from those societies which have been diverse in the past. Given the close relation between culture and religion, this will mean accommodation of diverse faiths as well as diverse cultures, while bearing in mind the host culture and religion.

It is well accepted that laws, which are widely ignored require to be reconsidered carefully, and such reconsideration may be very difficult to face up to. The fact that real (non-European) multi-cultural diversity is comparatively new in Western countries means that issues of this type are being considered for the first time. Such changes are naturally difficult and perhaps should be considered as part of normal evolution of society and not be laid exclusively at the feet of religion.

We reject the contrary view of a revealed religion and in place explain our own view of religion as tied to a reasoned view of what the moral law entails. [But see Chapter 5].

ENDNOTES

1. Peter Heather, 2006, *The Fall of the Roman Empire*, Pan: London.

2. Erica Beecher-Monas, 2007, *Evaluating Scientific Evidence*, Cambridge: Cambridge University Press. However undue reliance on the uncorroborated evidence or opinion of an expert witness can be dangerous or misleading.

3. *Holy Bible*, King James Version, St John18: 38.

4. In civil cases it is interesting to note the remarks of one of the great judges of his generation, Lord Denning, the Master of the Rolls, who said that the more important the matter, the stricter was his standard of the balance of probabilities.

5. William Twining, 2006, *Rethinking Evidence*, Cambridge: Cambridge University Press.

6. http://thisoxfordshire.co.uk/search/1407772.Mosque_sermon_sparks_complaints, 2007.

7. http://thisoxfordshire.co.uk/search/3858333.More_inter_faith_services_planned, 2008.

8. Philip Jenkins, 2008, *The Lost History of Christianity*, New York: Harper Collins.

9. Sidney H. Griffith, 2008, *The Church in the Shadow of the Mosque*, Princeton: Princeton University Press.

10. William Dalrymple,1998 From the *Holy Mountain*, London: Flamingo.

11. The way people turn to religion, either individually or *en masse*, in time of trouble or disaster, and find great solace in it, may be evidence of this assertion.

12. Garrett Mattingly, 2000 ,*The defeat of the Spanish Armada*, London: Pimlico.

13. Bernard Lewis, 1996, *The Middle East*, London: Phoenix.

14. H. Patrick Glenn, 2007, *Legal Traditions of the World*, Oxford: Oxford University Press.

CHAPTER 7 – EPILOGUE

So there you have it. Our own view on religion takes in the idea that we must have tolerance of other views. In Chapter 4 we have argued that Dennett is wrong in dismissing the role of Gödel's Theorem, and in Chapter 5 our own considered view may allow the possibility of Divine Action. On balance we have argued for a religious view and we have achieved this with argumentation, not mere rhetoric.

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