Quantum Field Theory and Philosophy.

QUANTUM FIELD THEORY and PHILOSOPHY. G.D.ROBERTSON.

A. Q.F.T. is an attempt to unify three older theories; field theory, quantum mechanics and special relativity, and provides a mathematical framework for describing elementary particle physics as will, I hope, become clear presently. Q.F.T. is not the only means used to examine elementary particles: strong interaction dynamics, current algebra and group theory (i.e. an examination of particle symmetries) are also important but these will, for the most part, be ignored in the following discussion.

As will be seen later no completely successful mathematical formulation of Q.F.T. has ever been produced, which is not surprising as quantum mechanics and relativity are in some respects contradictory.

1. In quantum mechanics the spatial variable, x, is an operator and the time variable, t, a c-number (or non-operator) while relativity requires them both to be of the same status. Q.F.T. manages to avoid this problem by making them both c-numbers.

2. The uncertainity principle shows that point particles can not exist in quantum mechanics as this would imply a well defined position could be associated with the particle. Relativity on the other hand can <u>only</u> admit point particles without which the principle requiring finite speed of transfer of information would be violated.(I am here assuming that a non-point particle has a rigid structure for without this assumption the particle could be forced to adopt an ellipsoidal shape of arbitrary length.)

In modern Q.F.T. particles are point-like and their experimentally observed structure is interpreted as some secondary dynamical effect. Now since these particles are point-like their eigen-energies turn out to be divergent and need renormalisation as described later. So in relativity there are well defined space-time boundaries while in quantum mechanics these boundaries can not be well defined. This leads to the suggestion that space and time may themselves be quantised yielding a minimum length, l°, and time, t°, the values of l° and t° being non-absolute but depending on the particular situation. This approach will possibly be able to avoid the contradictions involved in Q.F.T. but time will not permit further discussion of this here.

Although different in the above respects, it should be rembered that quantum mechanics and relativity are very similar in other respects; for example, at the foundations of both theories are to be found fundamental constants of nature, h and 1/c, which, if made to approach zero (i.e. $\hbar w/E$ and v^2/c^2 become negligible) yield classical physics from both theories. Since these are the two most basic physical theories available at present there is good reason to suppose that their unification would not only be desirable but necessary to obtain a more complete and conceptually accurate view of nature.

One example of Q.F.T.'s greater conceptual accuracy is found in wave-particle duality. Quantum mechanics suggests that not only should <u>all</u> elementary objects be regarded as having sometimes wave and sometimes particle properties but also that two apparently quite different objects (e.g. photons and electrons) should be treated as being at least very similar. Quantum mechanics cannot cope with this as it assumes particle creation and annihilation to be impossible except for particles with zero rest mass, ignoring pair creation, β-decay etc.... Thus photons must be treated differently from electrons. However, although in quantum mechanics the number of particles of finite rest mass present is a constant, in Q.F.T. the number of particles is a dynamical variable represented by an operator and thus the symmetry between photons and electrons is restored. It is hardly surprising, since particle production is a high energy process, that relativity should be required.

It may seem natural to suppose that in the limit of v^2/c^2 becoming negligible Q.F.T. may turn into quantum mechanics, but the situation is more complicated than this. Quantum mechanics is <u>not</u> a limiting case of Q.F.T. and so the concepts involved in quantum mechanics may not entirely be superseded by those of Q.F.T. A solution to this problem must wait for a more satisfactory formulation of Q.F.T.

B. I shall now attempt to outline the source of the relativistic wave equations for both particles with integral and half integral spin (bosons and fermions) and later show how they lead to a particle interpretation.

Firstly the equation for bosons is simply obtained by applying the quantum mechanical operators $\underline{p} \to -i\underline{\nabla}$, $E \to i\partial/\partial x_0$ to the relativistic energy equation $E^2 - \underline{p}^2 = m^2$. Writing everything in terms of 4-vectors gives $p^\mu \to i\partial^\mu$ and $p^2 = m^2$. ($p^2 = p^\mu p^\nu g_{\mu\nu}$) So $-\partial^2 = m^2$

$$\text{i.e.} \qquad (\partial^2 + m^2)\,\phi(x) = 0$$

This is called the Klein-Gordon Equation.

The corresponding equation for quantum electrodynamics is simply obtained from the above by substituting m=0 and $\emptyset \rightarrow A^{\mu}$ giving $\partial^2 A^{\mu} = 0$ where A^{μ} is the 4-vector potential ($\emptyset'$, $\underline{A}$).

The equation for spin $\frac{1}{2}$ particles is found from the time dependent Schrödinger equation $i\,\partial/\partial t\ \psi = H\ \psi$ by linearising it with respect to space and time (i.e. H contains $\underline{\nabla}$) and then requiring it to be invariant under rotation -- this second requirement forces the introduction of matrices and spinors. This gives $(\gamma . p)\,\psi(x) = m\ \psi(x)$

i.e. $(i\gamma . \partial - m)\,\psi(x) = 0$

which is called the Dirac Equation.

The Klein-Gordon equation has two solutions, one corresponding to the particle the other to the antiparticle while the Dirac equation has four solutions, two for each particle as they are of spin $\frac{1}{2}$ and therefore each have two possible spin orientations.

A note on the T.C.P. theorem.

There is a theorem called the T.C.P. theorem which states that, for relativistically invariant local field theories, invariance under time reversal, T, is equivalent to invariance under space inversion and charge conjugation taken together, CP. Taking the boson field as an example the following is a simplified proof of the theorem:

$\emptyset(x) \xrightarrow{P} \eta_P\, \emptyset(x^\circ, -\underline{x})$

$\emptyset(x) \xrightarrow{C} \eta_c\, \emptyset^{\dagger}(x)$

$\emptyset(x) \xrightarrow{T} \eta_T^*\, \emptyset(-x^\circ, \underline{x})$

$\emptyset(x) \xrightarrow{T.C.P.} \eta_P^* \eta_c^* \eta_T^*\, \emptyset^{\dagger}(-x)$

We can take $\eta_P^* \eta_c^* \eta_T^* = 1$. Now for a Lagrangian $L(x) = g\,\emptyset^3$:

$L(x) \xrightarrow{T.C.P.} L(-x)$. So when L is symmetrised with respect to the boson field $\emptyset$ it is invariant under T.C.P. i.e. $g\,\emptyset^3 + g^*\emptyset^{\dagger 3} \xrightarrow{C.P.T.} g^*\emptyset^{\dagger 3} + g\,\emptyset^3$.

We are considering the Lagrangian as any scattering process is dependent on the Lagrangian rather than the field or Hamiltonian as will be seen later.

Now in certain reactions, particularily $K_L \rightarrow 2\pi$, it is believed that CP is violated and so, by the T.C.P. theorem, T is also violated. From this asymmetry we can associate a direction with time at a fundamental level in physics. Note that for electromagnetic radiation there is invariance of T so, if the universe contained only radiation, which indeed it has done for most of its life since its explosion from a singularity about 10^{10} years ago, then no time direction can be found from this source. It is unclear at present what the connection between this source of a direction in time and those derived from thermodynamics or cosmology, if indeed there is any connection at all.

One further point to be noticed about the T.C.P. theorem is that it universally relates space, time and matter showing that in future none of these concepts can properly be discussed in isolation but only in connection with the other pair. (c.f. In a paper on relativity written in 1908 Minkowski made a si~~m~~milar claim: " Henceforth space by itself, and time by itself, are doomed to fade away into mere shaddows, and only a kind of union of the two will preserve an independent reality.")

C. A particle interpretation may be obtained from the above by Fourier Transforming the field function which is treated as an operator....

$$\phi(x) = 1/(2\pi)^3 \int d^3p \; (a(p) \, e^{-ip.x} + a^{+}(p) \, e^{ip.x}) \; / \; 2E \quad .$$

$a^{+}(p)$ may now be regarded as a creation operator for a particle of momentum p , while a(p) is an an~~x~~nihilation operator. i.e. $a^{+}(p) \,|\, 0 \rangle = |\, p \rangle$ and $a(p) \,|\, p \rangle = |\, 0 \rangle$.

Certain commutation relations must be imposed on these creation and annihilation operators to satisfy the particle statistics, in this case Bose-Einstein, and the uncertainty principle: $[a^{\dagger}(p)\ ,\ a^{\dagger}(p')] = 0$, $[a(p)\ ,\ a(p')] = 0$ and $[a(p)\ ,\ a^{\dagger}(p')] = (2\pi)^3\, 2E\ \delta^3(p-p')$.

Similarily for fermions, Fourier Transformation of the Dirac spinor gives....

$\psi(x) = 1/(2\pi)^3 \int d^3p\ \ (\ a(p,s)\ u_s(p)\ e^{-ip.x} + b^{\dagger}(p,s)\ v_s(p)\ e^{ip.x}\ )/2E$,

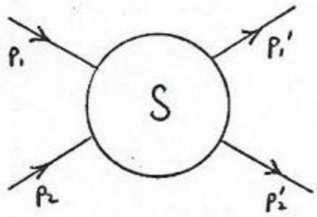
and again we impose relations (in this case anticommutation relations) on the creation and annihilation operators but this time such that the particles will obey Fermi-Dirac statistics $\{a\ ,\ b\} = \{a^{\dagger}\ ,\ b^{\dagger}\} = \{a\ ,\ b^{\dagger}\} = \{a^{\dagger}\ ,\ b\} = 0$ and $\{a(p,s),\ a^{\dagger}(p',s')\} = \delta_{ss'}\,(2\pi)^3\,\delta(p-p')2E$ etc... In the following we shall consider mainly boson fields as they are easier to work with.

Thus, for example, if we are considering electrons then $\psi(x)$ will correspond to the electron field which must be quantised to give the individual electrons, hence the name quantum field theory.

D. If we are to allow any interactions at all between fields, which is the goal of Q.F.T., then an interaction contribution must be added to the original Hamiltonian giving $H = H_o + H_I$ and a corresponding new Lagrangian $L = L_o + L_I$. Without L_I no scattering can take place since the scattering matrix is dependent on L_I as shown below.

During scattering the wave function changes according to $|\phi(t)\rangle = U(t)\,|\phi(-\infty)\rangle$. Now the scattering matrix is given by the value of $U(t)$ when we consider asymptotic times; $S = \lim_{t\to+\infty} U(t)$. Further calculation shows that this gives $S = T\ e^{i\int d^4x_i L_I(x_i)}$.

For this discussion we shall take the simplest possible sort of interaction Lagrangian $L_I(x) = g\,\phi^3$ (ϕ^2 just changes the mass of the particle as $L_0 = \left(\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 \right)$). Note that instead of the straightforward Klein-Gordon equation we are now solving $(\Box + m^2)\,\phi(x) = g\,\phi^3(x)$.
Now the S-matrix can be expanded in terms of ϕ into $S = S_0 + S_1 + S_2 + \ldots$ where $S_0 = I$, the unit matrix.

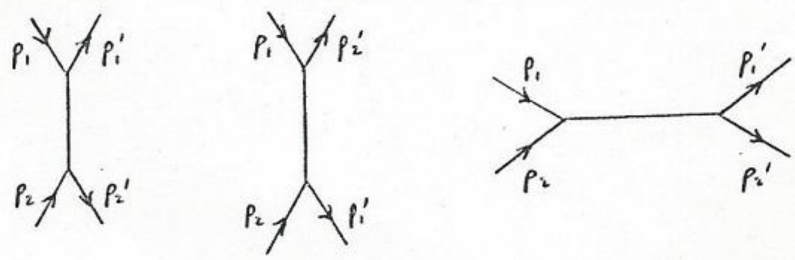


Applying this to the simple scattering process above and calculating the matrix elements for the reaction we find :
$\langle p_1'\ p_2' \mid S_0 \mid p_1\, p_2 \rangle = \langle 0 \mid a(p_1')\ a(p_2')\ a^{\dagger}(p_1)\ a^{\dagger}(p_2) \mid 0 \rangle\ /2$ (this gives no scattering). Similarily,

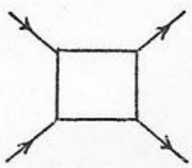
$$S_1 \longrightarrow ig\int d^4x \,\langle p_1'\ p_2' \mid \phi^3(x) \mid p_1\, p_2 \rangle \longrightarrow 0 \qquad \text{i.e. no contribution.}$$

$$S_2 \longrightarrow (ig)^2/2! \int d^4x\ d^4y \,\langle p_1'\ p_2' \mid T(\phi^3(x),\phi^3(y)) \mid p_1\, p_2 \rangle$$
$$\longrightarrow i(2\pi)^4\,\delta^4(P_f - P_i)\ 1/2! \left(-\frac{G^2}{s-m^2} - \frac{G^2}{t-m^2} - \frac{G^2}{u-m^2} \right)$$

These three parts correspond to diagrams of the form :

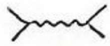


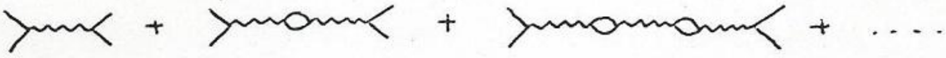
S_3 etc give more complex contributions e.g.



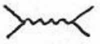
So for each topologically different diagram we get a different contribution to the S-matrix, and this new contribution corresponds approximately to a different type of scattering (Coulomb, Compton, Thompson, Rutherford, etc..) . It is possible to find a simple set of rules which, when applied to any diagram, give the S-matrix element for that diagram. This will prove useful later when the elements for very complex diagrams will be required.

The possible types of interactions between particles are limited by the conservation laws. Baryons and leptons must be conserved, charge conservation must be ensured, etc... However the uncertainty principle indicates that, without violating energy conservation, large ammounts of energy can become available for short periods, the larger the ammount the shorter the period. Thus particles may be produced from others (and from 'nothing') in as great an abundance as one cares for short intervals of time as long as the other conservation laws are observed. From this it follows that the container view of particles is quite untenable. Consider for example reactions of the type $A \rightarrow A + B + \overline{B}$ and $B \rightarrow B + A + \overline{A}$ which can both occur. Now it is clearly incorrect to suppose that A contained B and $\overline{B}$ in some sense before they were produced from A as this would contradict a similar account of the second reaction. So Q.F.T. assumes that particles can be created or destroyed without the necessity of their preexistence or postexistence. Such theories also require that if $A + B \rightarrow A + B$ is possible (i.e. some sort of scattering can take place) then $A \rightarrow A + B + \overline{B}$ is also possible. Thus any particle able to interact with our measuring instruments can be created from our familiar particles.

E. In this section we shall consider quantum electrodynamics (interaction of electrons and photons) as an example as it is the most complete and best understood part of Q.F.T. From the discussion above it is clearly possible for any photon line in a Feynman diagram to contain any number of electron bubbles  . (Vacuum Polarisation.) So instead of simply considering the effect of  in an interaction one must consider the effects of every member of an infinite series



The rules for calculating scattering elements produce the series;

$-g^{\mu\nu}/k^2 + (-i)\delta^{\mu}_{\nu'}/k^2 \Pi^{\mu'\nu'}(-i)\delta^{\nu}_{\mu'}/k^2 + \dots\dots$ which when summed together yield $-i\, Z_3\, g^{\mu\nu}/k^2$. So the effect of this series is to add a factor Z_3 to the element of the first diagram  . The presence of this term Z_3 shows that the measured physical charge on the electron can be written $e_{phys} = \sqrt{Z_3}\, e_{bare}$ where e_{bare} is the value of the charge on the electron which would be found if these 'virtual' processes above could not take place.

Similarily bubbles are allowed in the electron lines producing another infinite series which must be summed.

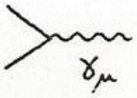
$$\frac{i}{\not{p}-m} + \frac{i}{\not{p}-m}(-i\Sigma)\frac{i}{\not{p}-m} + \dots\dots$$

$$\frac{i}{\not{p}-m-\Sigma(p)} \qquad \frac{i\, Z_2}{\not{p}-(m+\delta m)}$$

There are two consequences to note from this : m has been replaced by

$m + \delta m$ (δm being the electromagnetic self energy) which is called mass renormalisation and each electron wave function u(p) in the field operator $\psi(x)$ has been replaced by $\sqrt{Z_2}\,u(p)$.

Yet another type of renormalisation, vertex renormalisation, must be considered which is simply the possibility of a photon passing between the two electron lines at a vertex:



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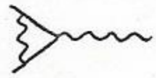
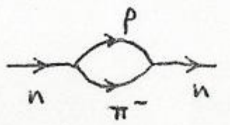


$\longrightarrow \frac{1}{Z_1}\gamma_\mu$

Now taking all these effects into account we find the physical (or measured) charge on the electron is given by $e_{phys} = (\sqrt{Z_3}\,Z_2 / Z_1)\, e_{bare}$ and all three Z's turn out to be infinite as the series are divergent. However an exact mathematical result known as Ward's Identity shows that $Z_1 = Z_2$ so the above simplfies to $e_{phys} = \sqrt{Z_3}\, e_{bare}$.

In order to remove the infinities and obtain results which can be experimentally verified, we replace the bare charge, mass etc.. in L_I by the corresponding physical values. Having done this, physical predictions turn out to be finite and in excellent agreement with observed results. Removal of explicit mention of these bare quantities is called renormalisation.

F. We have seen how, in the above, the behaviour of a system in Q.F.T. is not only influnced by its constituents and surroundings but also by any system that could exist under the same conditions. ('Could' is here intended to be taken in the sense of physical possibility.) For example, it is possible for a neutron to spontaneously become a proton and a pion of negative charge although only for a short time if violation of conservation is to be avoided.



A neutron has zero charge and would therefore be expected to have zero magnetic moment, however experimentally it is found that a neutron has a finite magnetic moment which is accounted for in Q.F.T. by taking into account the possibility that it will decay into $p^{+} + \pi^{-}$, both of which <u>have</u> charge. Thus it is found that apparently simple particles can have much more complicated properties than expected and it is this fact that has hindered the mathematical development of Q.F.T.

Are we then to suppose that something which is only a possibility and not in some sense a reality can influence actual measurements? Taking this view assumes we are approaching the end of physics by assuming that what are at present termed elementary particles are indeed fundamental. Now since at present at least 500 such particles have been discovered this assumption seems to be quite unfounded.

To avoid the unpleasant assumption that there exist 500 equally fundamental particles, a modern approach has been to introduce a small number of new fundamental particles (called quarks) which combine in different ways to give all the strongly interacting particles or hadrons. Bosons result from the combination of a quark and an antiquark while baryons contain three quarks or three antiquarks. All other combinations, termed exotics, are presumed not to exist.

It seems likely therefore, when more of the physics of elementary particles is understood, the notion of mere possibility will be discarded in favour of some sort of reality. A plausible idea suggested by the above is that the virtual states which we have been discussing have the same ontological status as the elementary particles which are quasi-stable states of energy in space-time.

G. The C.P.T. theorem and so the Feynman diagram representation indicate that there are alternative ways of describing a particle: either one can call it a particle travelling forward in time and space or one can call it an antiparticle travelling backwards in space and also in time.

Renormalisation theory indicated that particle properties depend not only on the particles themselves but also on all the properties of many other particles and further that these particles may only be quasi-stable states of energy in space-time, jumping from one quasi-stable state to another.

All these aspects of Q.F.T. suggest that space, time and matter are inseparably related ideas, as does that other giant sized physical theory, General Relativity, but this whole topic of Generalised Q.F.T. is outside the scope of this discussion.

So the final picture we have is as follows:'Particles' are only individual particles on measurement, the rest of the time they are rapidly changing from one quasi-stable state of a cloud of virtual particles to another. Thus an explanation of many of the conceptual dilemmas of the less general quantum mechanics may be afforded. For example the two slit experiment requires no simple trajectory of one particle from the source to the observer and therefore does'nt require the unsatisfactory notion of wave-particle duality.

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